Examen Parcial # 1. CÁLCULO INTEGRAL. Martes, 21, Mayo, 201

(Soluciones) Universidad Autónoma Metropolitana - Azcapotzalco

(1) (a) $\Delta x = \frac{b-a}{n} = \frac{4-0}{n} \Rightarrow \boxed{\Delta x = \frac{4}{n}}$, and $x_k = x_0 + k\Delta x$

$= 0 + k\Delta x$

ie $\boxed{x_k = k\Delta x}$

The Riemann sum is:

$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} 2x_k\,\Delta x = 2\sum_{k=1}^{n} k(\Delta x)^2 = 2(\Delta x)^2\sum_{k=1}^{n} k$$

Δx is k-independent

$$= 2\left(\frac{4}{n}\right)^2 \frac{n(n+1)}{2} \quad \cdots\cdots (*)$$

Hence:

$$\sum_{k=1}^{n} f(x_k)\Delta x = 2\left(\frac{4}{n}\right)^2 \frac{n(n+1)}{2} = 4^2\,\frac{n}{n}\,\frac{(n+1)}{n}$$

and

$$(\text{Area})_1 = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x = 4^2 \lim_{n\to\infty}\frac{n}{n}\left(1+\frac{1}{n}\right) = 4^2\cdot 1\cdot 1$$

$$\boxed{(\text{Area})_1 = 16}$$

(b) Using the previous argument, $\Delta x = \frac{b-a}{n} = \frac{2-0}{n} \Rightarrow \Delta x = \frac{2}{n}$

Using ~~the~~ formula $(*)$, with $\Delta x = \frac{2}{n}$:

$$\sum_{k=1}^{n} f(x_k)\,\Delta x = 2\left(\frac{2}{n}\right)^2 \frac{n(n+1)}{2}$$

Then:

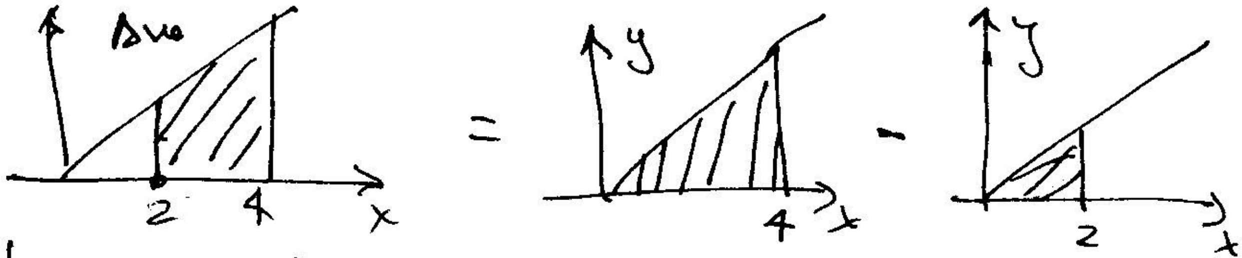
$$(\text{Area})_2 = \lim_{n\to\infty} 2\left(\frac{2}{n}\right)^2 \frac{n(n+1)}{2} = \lim_{n\to\infty} 2^2\,\frac{n}{n}\left(\frac{n+1}{n}\right)$$

$$= 4\cdot 1\cdot 1$$

$\Rightarrow (\text{Area})_2$.

(c) Now

By the properties of definite integrals



$$(\text{Area})_3 = (\text{Area})_1 - (\text{Area})_2$$

Hence

$$A_3 = A_1 - A_2 = 16 - 4 = 12$$

$$\boxed{(\text{Area})_3 = 12}$$

$$\int_2^4 f(x)\,dx = \int_0^4 f(x)\,dx - \int_0^2 f(x)\,dx$$ } Property of the integral

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(2) (a) $\dfrac{df}{dx} = 3t^2 \Big|_{t=\sin x} \cdot \dfrac{d}{dx} \sin x$, by the chain rule, and the Fundamental Theorem of Calculus.

$$= 3(\sin x)^2 \cdot \cos x$$

(b) Compute antiderivative:

(b) $f(x) = \displaystyle\int_1^{\sin x} 3t^2\,dt = t^3 \Big|_1^{\sin x} = \sin^3 x - 1$

By the Fundamental Theorem of Calculus

$$\Rightarrow f'(x) = 3\sin^2 x \cos x$$

(c) We found the same result, since part (a) is the Part A of the Fundamental Theorem of Calculus, while part (b) is the Part B of The Fundamental Theorem of Calculus.

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③ (a) $\int 6x^2 \cos x^3 \sin x^3 \, dx = \int 6x^2 \cos y \sin y \frac{dy}{3x^2} =$

where $y = x^3$

$dy = 3x^2 \, dx$.

$= \int 2 \cos y \sin y \, dy = \int 2z \, dz = z^2 + C$

$z = \sin y$

$dz = \cos y \, dy$

$= (\sin y)^2 + C$

$= \sin^2(x^3) + C$

Hence $\boxed{\int 6 x^2 \cos^3 \sin x^3 \, dx = \sin^2(x^3) + C}$

(b) $\int_1^4 \frac{10\sqrt{x}}{(1+x^{3/2})^2} dx = 10 \int_\alpha^\beta \frac{\frac{2}{3} du}{(1+u)^2} = \frac{20}{3} \int_1^8 \frac{du}{(1+u)^2}$

$\left[\begin{array}{l} u = x^{3/2} \\ du = \frac{3}{2} x^{1/2} dx, \text{ i.e. } \frac{2}{3} du = x^{1/2} dx \end{array} \right]$

and $\left[\begin{array}{l} \cancel{x} = 1 \Rightarrow y = \alpha = (\sqrt{1})^3 = 1 \\ x = 4 \Rightarrow y = \beta = 4^{3/2} = (\sqrt{4})^3 = 2^3 = 8 \end{array} \right]$

$= \frac{20}{3} \left(\frac{-1}{1+u} \right) \Big|_1^8 = \frac{20}{3} \left(\frac{-1}{1+8} + \frac{1}{1+1} \right) = \frac{20}{3} \frac{7}{18} =$

(4) (a) $\int x^2 e^x\,dx = x^2 e^x - \int 2x e^x\,dx$, integration by parts

$= x^2 e^x - 2x e^x + \int 2e^x\,dx$, by integration by parts

$= x^2 e^x - 2x e^x + 2e^x + C$

i.e. $$\boxed{\int x^2 e^x\,dx = (x^2 - 2x + 2)e^x + C}$$

(b) $\int x \cos 3x\,dx = \frac{1}{3} x \sin 3x - \frac{1}{3}\int \sin 3x\,dx$, by parts

$$\boxed{\int x \cos 3x\,dx = \frac{1}{3} x \sin 3x + \frac{1}{3^2} \cos 3x + C.}$$

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(5) Since the integrand is an even function,

$$\int_{+100}^{-100} x^3 \cos x\, e^{-x^4} \log(1+x^8) \sin^2 x\,dx = \underline{\underline{0}}$$

the integral is <u>zero</u>
