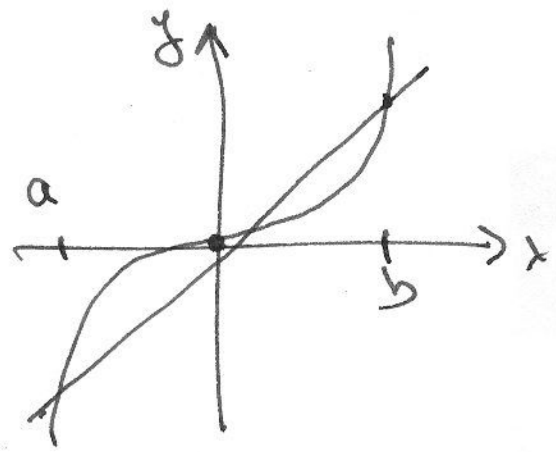
(1) Encontremos los puntos de intersección: 04 Julio 2013.

Tercer Examen Parcial
Cálculo Integral
UAM-A.
Spring 2013



$$x^3 = x$$

$$x^3 - x = 0$$

$$x(x^2-1) = 0$$

$$x(x-1)(x+1) = 0 \Rightarrow \begin{cases} x = -1 \\ x = 0 \\ x = +1 \end{cases}$$

límites de integración: $a = -1$, $b = +1$

$$\text{Area} = \int_a^b |f(x) - g(x)|\,dx = \int_{-1}^{1} |x^3 - x|\,dx =$$

$$= \int_{-1}^{0} (x^3 - x)\,dx + \int_0^1 (x - x^3)\,dx$$

pues $x^3 > x$ en $[-1,0]$

pues $x > x^3$ en $[0,1$

$$= \left(\frac{x^4}{4} - \frac{x^2}{2}\right)\Big|_{-1}^{0} + \left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_0^1$$

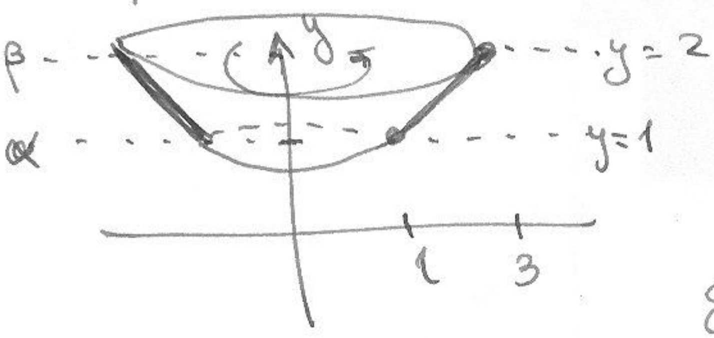
$$= \left\{0 - \left(\frac{1}{4} - \frac{1}{2}\right)\right\} + \left\{\left(\frac{1}{2} - \frac{1}{4}\right) - 0\right\}$$

$$= 2\left(\frac{1}{2} - \frac{1}{4}\right) = 2 \cdot \frac{1}{4} \Rightarrow \boxed{\text{Area} = \frac{1}{2}\,(\text{unidades})^2}$$

② Aquí tenemos la función:

$$y = \frac{x}{2} + \frac{1}{2} \quad , \text{con} \quad 1 \leq x \leq 3 .$$

Pero ahora consideremos x como función de y pues la rotación es alrededor del eje y:



Si $x = 1$, $\Rightarrow y = 1$

Si $x = 3 \Rightarrow y = 2$

Entonces $\alpha = 1$

$\beta = 2$

Además

$$y = \frac{x}{2} + \frac{1}{2} \Rightarrow 2y = x + 1 \Rightarrow 2y - 1 = x$$

$$\Rightarrow \quad x = 2y - 1$$

Aquí $g(y) = 2y - 1$, $\Rightarrow g'(y) = 2$

La fórmula es:

$$A = 2\pi \int_{\alpha}^{\beta} g(y) \sqrt{1 + (g'(y))^2}\, dy$$

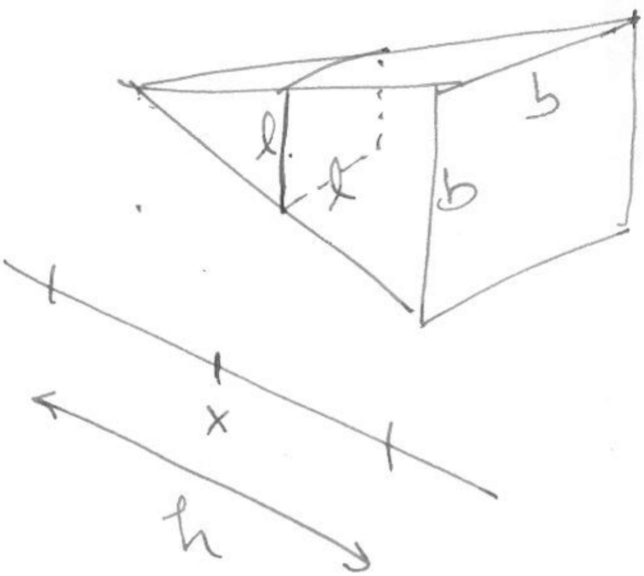
Entonces:

$$A = 2\pi \int_1^2 (2y-1)\sqrt{1 + 2^2}\, dy = 2\pi\sqrt{5} \int_1^2 (2y-1)\, dy$$

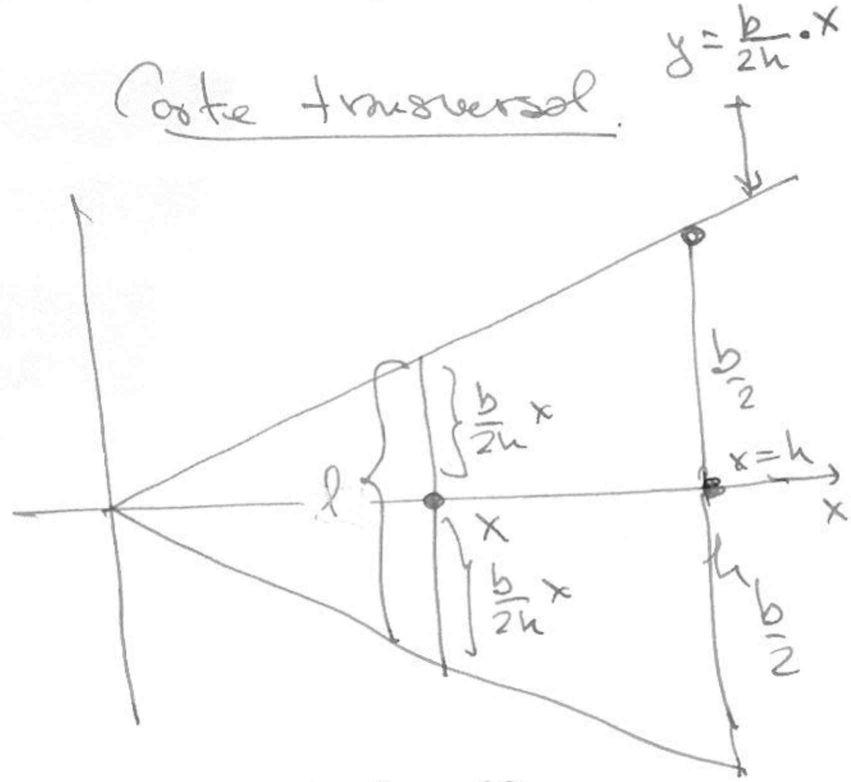
$$= 2\pi\sqrt{5}\, \left(y^2 - y\right)\Big|_1^2 = 2\pi\sqrt{5}\left[(2^2-2) - (1^2-1)\right]$$

$$= 2\pi\sqrt{5}\left[(4-2) - 0\right] \Rightarrow \boxed{\text{Area} = 4\pi\sqrt{5}}$$

③ Calcule el área de la pirámide de base b, y altura h (Aquí b=3, h=3)



Corte transversal



Area de sección transversal: $A(x) = l^2$.

Pero $l = \frac{b}{2h}x + \frac{b}{2h}x = \frac{b}{h}x$.

Entonces:

$$A(x) = \left(\frac{b}{h}x\right)^2.$$

$$\text{Volumen} = \int_0^h A(x)\,dx = \int_0^h \frac{b^2}{h^2}x^2\,dx = \frac{b^2}{h^2}\frac{x^3}{3}\Big|_0^h$$

$$= \frac{b^2}{h^2}\frac{1}{3}(h^3 - 0^3) = \frac{b^2}{3h^2}h^3 = \frac{b^2 h}{3}$$

$\boxed{\text{Volumen} = \frac{b^2 h}{3}}$ y es el volumen conocido de una pirámide.

Aquí $b=3, h=3 \Longrightarrow \boxed{\text{Volumen} = \frac{3^2 \cdot 3}{3} = 9}$

= 3 =

④ Longitud de la gráfica de

$$y = x^{3/2}$$

de $x = 0$ a $x = 4$.

Usemos la fórmula:

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

Aquí: $f(x) = x^{3/2} \Rightarrow f'(x) = \frac{3}{2} x^{1/2}$

$$\Rightarrow (f'(x))^2 = \frac{9}{2} x$$

Entonces:

$$L = \int_a^b \sqrt{1 + \frac{9}{4} x}\, dx = \int_0^4 \left[1 + \frac{9}{4} x\right]^{1/2} dx = \frac{2}{3} \left.\frac{\left(1 + \frac{9}{4} x\right)^{3/2}}{\frac{9}{4}}\right|_0^4$$

$$= \frac{8}{27} \left(1 + \frac{9}{4} x\right)^{3/2} \Big|_0^4 = \frac{8}{27} \left[\left(1 + \frac{9 \cdot 4}{4}\right)^{3/2} - 1^{3/2}\right]$$

$$= \frac{8}{27} \left(10^{3/2} - 1\right)$$

$$\Rightarrow \boxed{L = \frac{8}{27} \left(10^{3/2} - 1\right)}$$

(5) Necesitamos, para empezar, calcular la constante de Hooke:

$$F = -kx.$$

Si $F = 2N$ y $x = 0.02\ m$:

$$\Rightarrow \quad 2N = k\,\frac{2}{100}\,m \Rightarrow \boxed{k = 100\ \frac{N}{m}}$$

$$\Rightarrow \quad \boxed{F = -100x}$$

(a) Si $F = 4N$, entonces

$$4 = 100 \cdot x \Rightarrow \boxed{x = \frac{4}{100}\ m} = 4\,cm$$

(b) Debemos calcular:

$$W = \int_0^{4/100} F(x)\,dx = -\int_0^{4/100} kx\,dx = -\frac{k}{2}x^2\Big|_0^{4/100}$$

$$= -\frac{k}{2}\left[\left(\frac{4}{100}\right)^2 - 0\right] = -\frac{k}{2}\frac{4^2}{(100)^2}$$

$$\Rightarrow \quad W = -\frac{k}{2}\frac{16}{(100)^2} = -\frac{100}{2}\cdot\frac{16}{100^2} = -\frac{16}{100}$$

$$\boxed{W = -\frac{16}{100}\ \text{Joules}}$$