UAM–Azcapotzalco Jueves, Octubre 27, 2016

Introducción al Cálculo. Examen #1-A

(1) (a) $-2x+1 < x-3 \Rightarrow 1+3 < 2x+x \Rightarrow 4 < 3x$

$\Rightarrow \frac{4}{3} < x \Rightarrow \boxed{x \in \left(\frac{4}{3}, \infty\right)}$

(b) $\frac{6}{x-1} \leq 5 \Rightarrow \frac{6}{x-1} - 5 \leq 0 \Rightarrow \frac{6-5x+5}{x-1} \leq 0$

$\Rightarrow \frac{11-5x}{x-1} \leq 0$.

Caso (a)

$\left.\begin{array}{l} 11-5x \leq 0 \\ x-1 > 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \frac{11}{5} \leq x \\ x > 1 \end{array}\right\} \Rightarrow \begin{array}{l} \frac{11}{5} \leq x \\ 1 < x \end{array} \Rightarrow x \in \left[\frac{11}{5}, \infty\right)$.

Caso (b)

$\left.\begin{array}{l} 11-5x \geq 0 \\ x-1 < 0 \end{array}\right\} \quad \left.\begin{array}{l} \frac{11}{5} \geq x \\ x < 1 \end{array}\right\} \Rightarrow \begin{array}{l} x < 1 \\ x \leq \frac{11}{5} \end{array} \Rightarrow x < 1 \Rightarrow x \in (-\infty, 1)$.

$\boxed{\text{Solution set} = (-\infty, 1) \cup \left[\frac{11}{5}, \infty\right)}$

(2) (a) $|x-1| = 5$

Case (i)

$\left.\begin{array}{l} x-1 \geq 0 \\ \Rightarrow x-1 = 5 \end{array}\right\} \Rightarrow \begin{array}{l} x > 1 \\ x = 6 \end{array} \Rightarrow \boxed{x_1 = 6}$

Solutions

Case (ii)

$\left.\begin{array}{l} x-1 < 0 \\ -(x-1) = 5 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x < 1 \\ -x+1 = 5 \end{array}\right\} \Rightarrow -x = 5-1 \Rightarrow \boxed{x_2 = -4}$

$\boxed{\begin{array}{l} x_1 = 6 \\ x_2 = -4 \end{array}}$

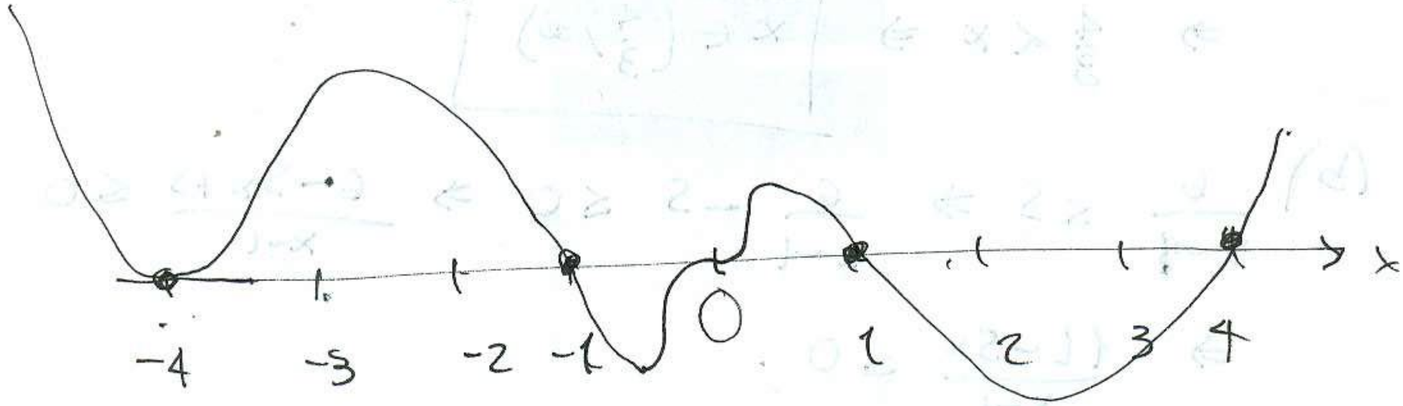
(b) $|x+2| = -1$.

Since $|x+2| \geq 0$, this would imply $-1 \geq 0$, which is false. $\Rightarrow$ There is no solution. $\boxed{\text{Solution set: } \phi \text{ the empty set}}$

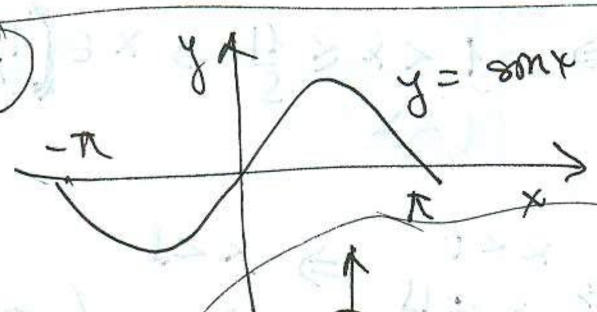
(3) $P(x) = (x+4)^2(x^2-1)\,x^3(x-4)$.

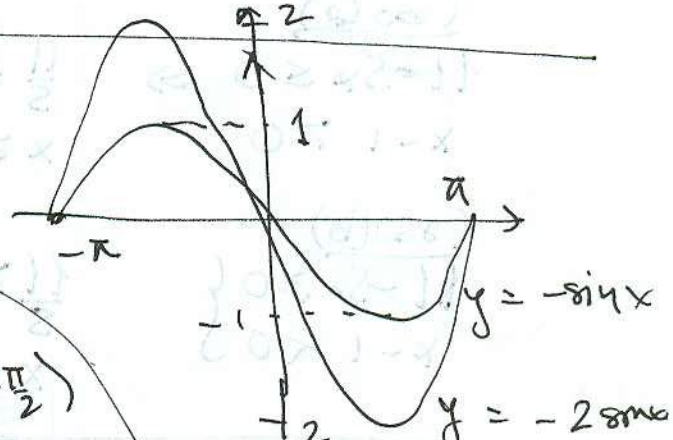
$= (x+4)^2(x-1)\,x^3(x+1)(x-4)$

$P(x)$ is a polynomial of degree 8.



(4)





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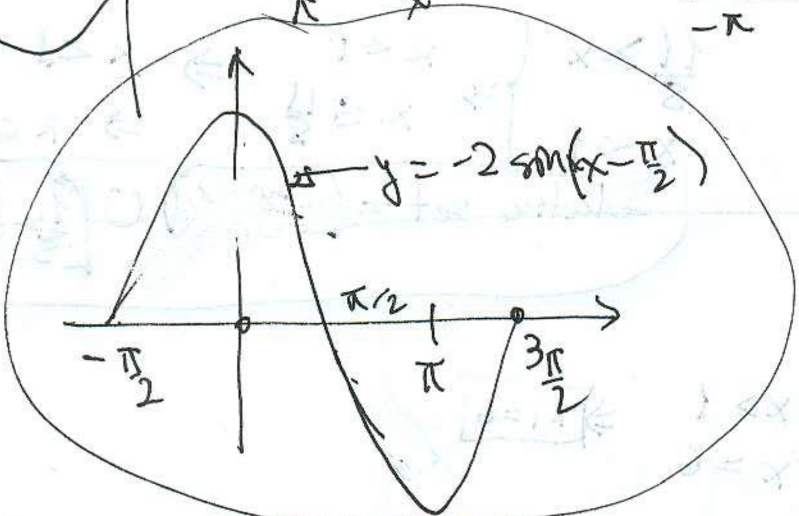


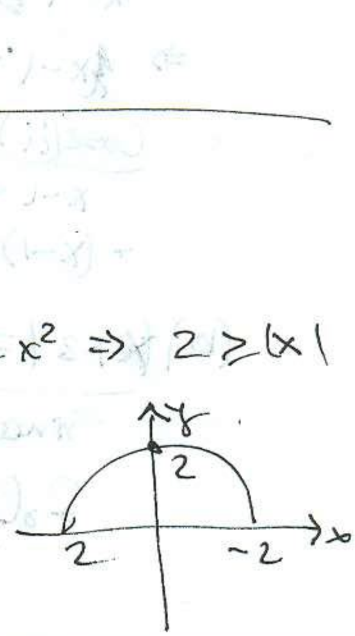
(5) $f(x) = \sqrt{4-x}$, $g(x) = x^2$.

(a) $(f\circ g)(x) = f(g(x)) = \sqrt{4-g(x)} = \sqrt{4-x^2}$

(b) $\mathrm{Dom}(f\circ g)$ is such that $4-x^2 \geq 0 \Rightarrow 4 \geq x^2 \Rightarrow 2 \geq |x|$

$\Rightarrow -2 \leq x \leq 2 \Rightarrow \boxed{\mathrm{Dom}(f\circ g) = [-2, 2]}$

$\mathrm{Rng}(f\circ g) = [0, 2]$, since it is a semicircle



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Introducción al Cálculo Examen # 1-B.

(1) (a) $-3x+1 < x-2 \Rightarrow 1+2 < 3x+x \Rightarrow 3 < 4x$

$\Rightarrow \frac{3}{4} < x \Rightarrow$ Conjunto solución $(\frac{3}{4}, \infty)$

(b) $\frac{5}{x+1} \le 6 \Rightarrow \frac{5}{x+1} - 6 \le 0 \Rightarrow \frac{5-6x-6}{x+1} \le 0$

$\Rightarrow \frac{-1-6x}{x+1} \le 0.$ Caso (a)

$\left.\begin{array}{l} -1-6x \le 0 \\ x+1 > 0 \end{array}\right\} \Rightarrow \begin{array}{l} -\frac{1}{6} \le x \\ x > -1 \end{array} \Rightarrow \begin{array}{l} -1 < x \\ -\frac{1}{6} \le x \end{array}$

$\Rightarrow x \in [-\frac{1}{6}, \infty)$

Conjunto solución

$\boxed{(-\infty, -1) \cup [-\frac{1}{6}, \infty)}$

Caso (b)

$\left.\begin{array}{l} -1-6x \ge 0 \\ x+1 < 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -\frac{1}{6} \ge x \\ x < -1 \end{array}\right\} \Rightarrow \begin{array}{l} x \le -\frac{1}{6} \\ x < -1 \end{array}$

$\Rightarrow x \in (-\infty, -1).$

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(2) (a) $|x+5| = 1.$

Caso (i) $x+5 \ge 0$

$x+5 = 1 \Rightarrow x = 1-5 \Rightarrow x = -4$

Caso (ii) $x+5 < 0$

$-(x+5) = 1 \Rightarrow x = -1-5 \Rightarrow x = -6$

Solutions
$x_1 = -4$
$x_2 = -6$

(b) $|x-1| = -2.$

Since $|x-1| \ge 0$ and $-2 < 0$, $|x-1| \neq -2$, always.

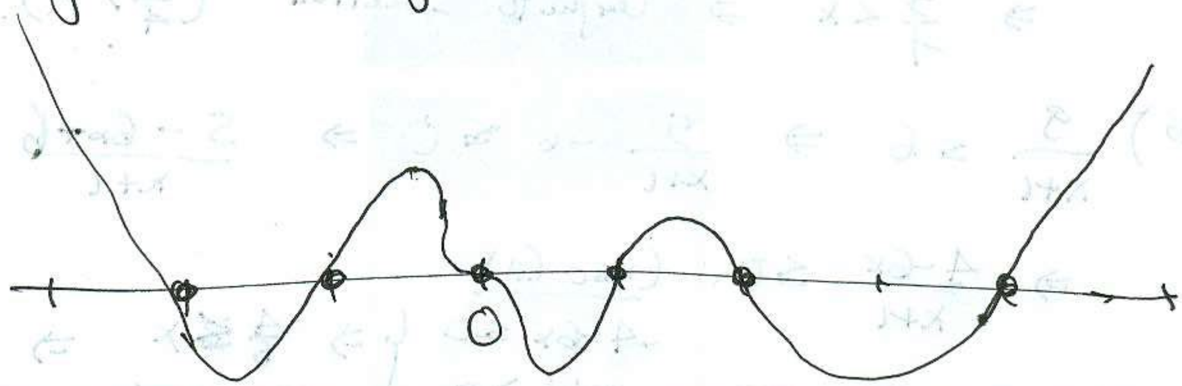
Then, there is no solution

The solution set is the empty set : ϕ

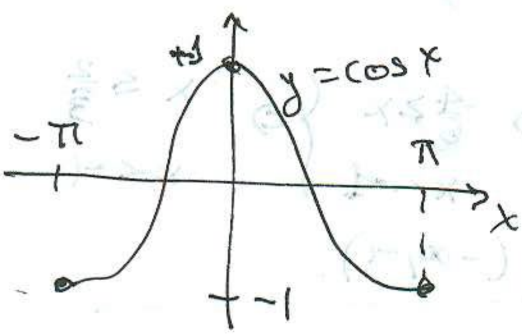
(3). $Q(x) = (x^2-4)(x^2-1)\,x^3\,(x-4)$

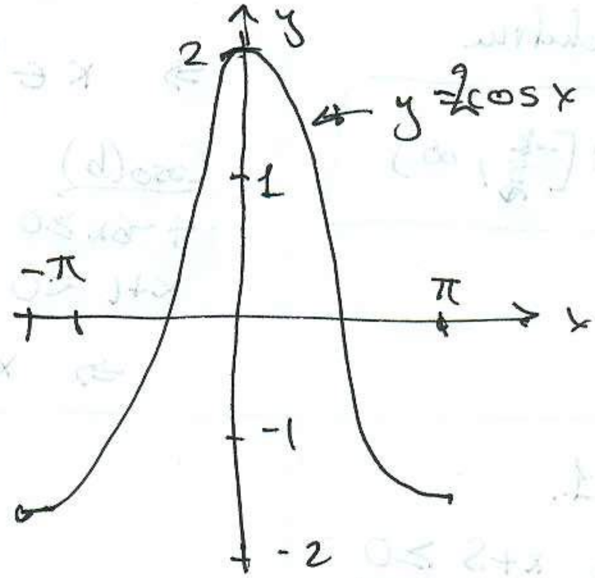
$= (x-2)(x+2)(x-1)(x+1)\,x^3\,(x-4)$

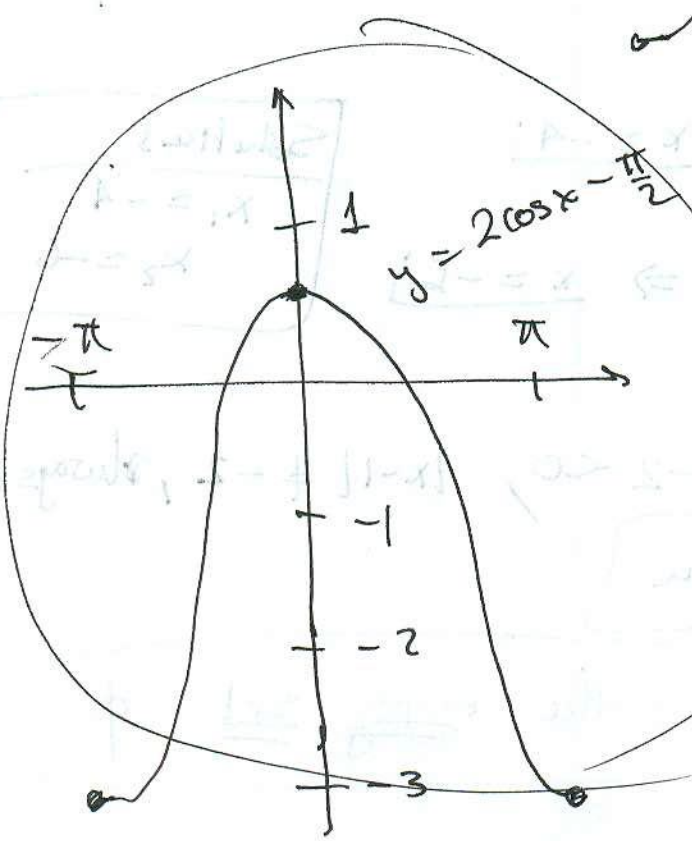
$Q(x)$ is Polynomial of degree 8.



(4) (a)







(5) (a) $(G \circ g)(x) = G(g(x))$

$= -g(x) = -\sqrt{25 - x^2}$

Domain $25 - x^2 \geq 0 \Rightarrow x^2 \leq 25$

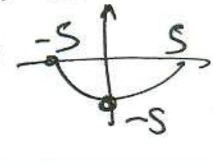
$\Rightarrow \sqrt{x^2} \leq 5 \Rightarrow |x| \leq 5$

$\Rightarrow -5 \leq x \leq 5$

$\text{Dom}(G \circ g)(x) = [-5, 5]$

$\text{Rang}(G \circ g)(x) = [-5, 0]$

Since $y = -\sqrt{25 - x^2}$



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Introducción al Cálculo. Examen #1 - C

(1) (a) $-x + 2 > 3x - 1$

$2 + 1 > 3x + x$

$3 > 4x$

$\frac{3}{4} > x$

$\boxed{x \in \left(-\infty, \frac{3}{4}\right)}$

(b) $\frac{6}{x-5} \geq 1$

$\frac{6}{x-5} - 1 \geq 0$

$\frac{6 - x + 5}{x-5} \geq 0 \Rightarrow \frac{11 - x}{x-5} \geq 0$

Caso (a) $11 - x \geq 0$, $x - 5 > 0$

$\Rightarrow 5 < x \leq 11 \Rightarrow x \in (5, 11]$.

Caso (b) $11 - x \leq 0$, $x - 5 < 0$ $\Rightarrow$

$\Rightarrow$ $11 \leq x$, $x < 5$ } Impossible

Solution:
$x \in (5, 11]$

(2) (a) $|x - 2| = 6$

Caso (a) $x - 2 \geq 0$.

$x - 2 = 6 \Rightarrow \boxed{x = 8}$

Caso (b) $x - 2 < 0$

$-(x-2) = 6 \Rightarrow -x + 2 = 6 \Rightarrow -x = 4 \Rightarrow \boxed{x = -4}$

Solutions $\boxed{\begin{array}{l} x_1 = 8 \\ x_2 = -4 \end{array}}$

(b) $|x^2 + 2| = -10$

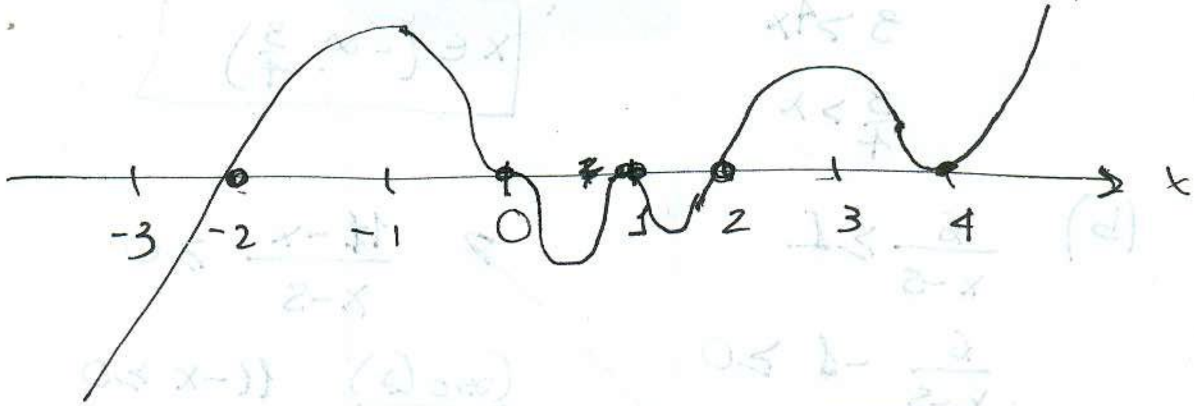
Since $|x^2 + 2| > 0$ and $-10 < 0$, this equation is incompatible. $\boxed{\text{There is } \underline{\text{no}} \text{ solutions}}$ the solution is the empty set: ϕ

3) Graph $R(x) = (x^2-4)(x-1)^2 x^3 (x-4)^2$.

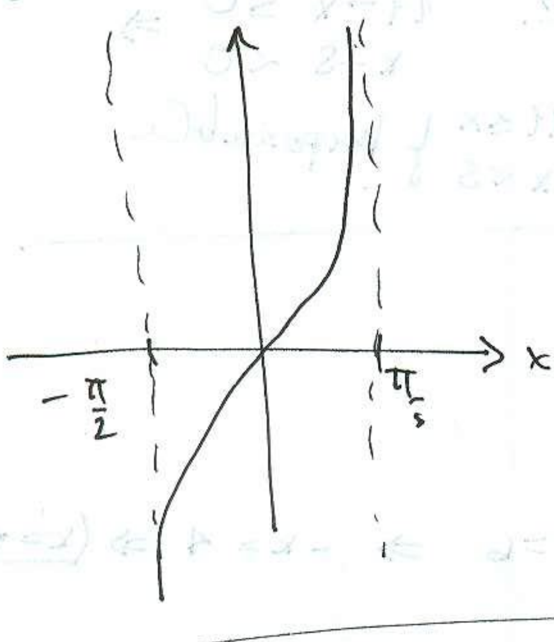
This is a polynomial of degree 9..

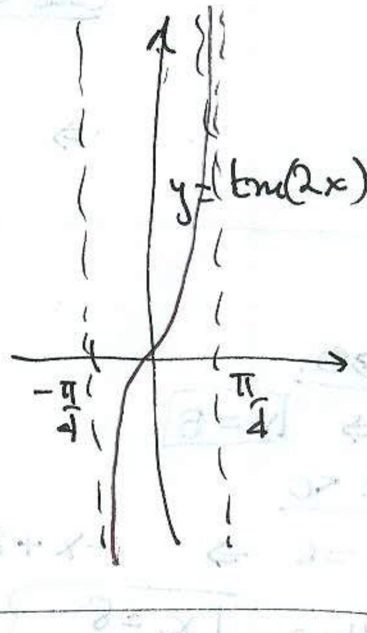
It can be re-written as:

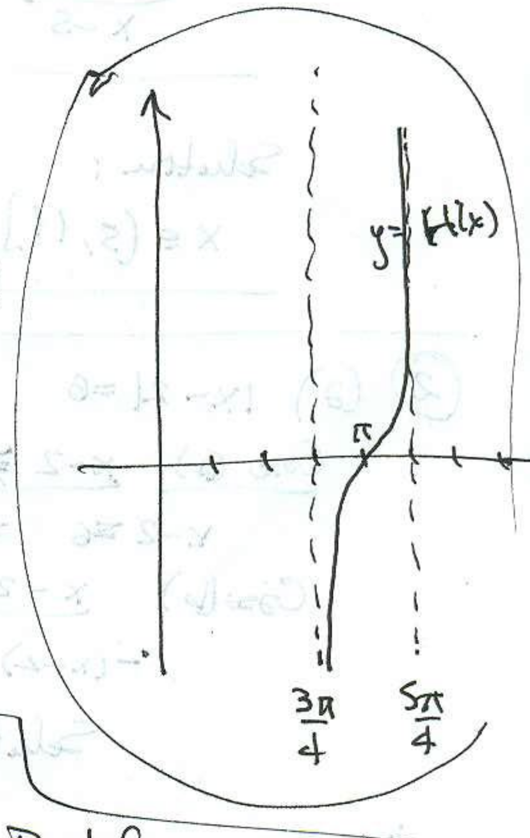
$$R(x) = (x-2)(x+2)(x-1)^2 x^3 (x-4)^2$$



4). $h(x) = \tan x$, on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$







5) (a) $(h \circ H)(x) = h(H(x)) = |H(x) + 1|$

$\Rightarrow (h \circ H)(x) = |x^2 + 1| = x^2 + 1$: Parabola

Dom $(h \circ H) = \mathbb{R}$

Ran $(h \circ H) = [1, \infty)$