Introducción al Cálculo | Viernes 25 de noviembre de 2016

Exam #2 - A | Universidad Autónoma Metropolitana - Azcapotzalco

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(1) $\lim_{x\to 0} \frac{3\tan(3x)}{x\sec(x)} = \lim_{x\to 0} 3 \frac{\sin(3x)}{x} \frac{\cos(x)}{\cos(3x)} = \lim 3^2 \frac{\sin(3x)}{(3x)} \frac{\cos x}{\cos(3x)}$

$= 3^2 \lim_{x\to 0} \frac{\sin(3x)}{3x} \frac{\cos x}{\cos(3x)}$, by properties of limits

$= 3^2 \lim_{x\to 0} \frac{\sin(3x)}{3x} \frac{\lim_{x\to 0}\cos x}{\lim_{x\to 0}\cos(3x)}$, by properties of limits.

$= 3^2 \cdot 1 \cdot \frac{1}{1}$, since $\lim_{y\to 0} \frac{\sin(y)}{y} = 1$.

Hence:

$$\boxed{\lim_{x\to 0} \frac{3\tan(3x)}{x\sec x} = 9.}$$

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(2). By trigonometric identity:

$$\cos(2\theta) = 2\cos^2\theta - 1.$$

$$\Rightarrow \quad \cos^2(\theta) = \frac{\cos(2\theta)+1}{2}$$

Take $\theta = \frac{\pi}{12}$. Hence

$$\cos^2\left(\frac{\pi}{12}\right) = \frac{\cos\left(\frac{\pi}{6}\right)+1}{2} = \frac{\frac{\sqrt{3}}{2}+1}{2}$$

$$= \frac{\sqrt{3}+2}{4}.$$

$$\Rightarrow \cos\left(\frac{\pi}{12}\right) = \pm\sqrt{\frac{\sqrt{3}+2}{4}}.$$

Now, $\frac{\pi}{12}$ is in the first quadrant. Then: $\cos\left(\frac{\pi}{12}\right) > 0$,

so, choose "+" sign.

$$\boxed{\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{\sqrt{3}+2}}{2}}$$

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(3) $$\lim_{x\to-\infty} \frac{\sqrt{x^2+4}}{x+2} = \lim_{x\to-\infty} \frac{\sqrt{x^2\left(1+\frac{4}{x^2}\right)}}{x\left(1+\frac{2}{x}\right)} =$$

$$= \lim_{x\to-\infty} \frac{\sqrt{x^2}\sqrt{1+\frac{4}{x^2}}}{x\left(1+\frac{2}{x}\right)} = \lim_{x\to-\infty} \frac{|x|\sqrt{1+4/x^2}}{x\left(1+2/x\right)}$$

Since $x \to -\infty \Rightarrow x < 0 \Rightarrow |x| = -x$. Hence:

$$= \lim_{x\to-\infty} \frac{-x}{x} \frac{\sqrt{1+4/x^2}}{(1+2/x)} = \lim_{x\to-\infty} \frac{(-1)\sqrt{1+4/x^2}}{(1+2/x)}$$

$$= \frac{(-1)\sqrt{1+0}}{\sqrt{1+0}} \Rightarrow \boxed{\lim_{x\to-\infty} \frac{\sqrt{x^2+4}}{x+2} = -1}$$

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(4) $$f(x) = \frac{2(x^2+x-6)}{x^2-3x+2} = \frac{2(x+3)(x-2)}{(x-1)(x-2)}$$

(a) $\text{Dom}(f) = \mathbb{R} \setminus \{1, 2\}$

(b) $f(0) = \frac{2(-6)}{2} = -6$ $\boxed{y=-6}$ Y-intersept.

$$f(x) = 0 \Rightarrow \frac{2(x+3)(x-2)}{(x-1)(x-2)} = 0 \Rightarrow \frac{2(x+3)}{(x-1)} = 0$$

since $x \neq 2$

$\Rightarrow \quad 2(x+3)=0$, since $x \neq 1$.

$\Rightarrow \quad \boxed{x=-3}$ x-intercept.

(c). It is not periodic:

$$f(-x) = \frac{2((-x)^2 - x - 6)}{(-x)^2 + 3x + 2} = \frac{2(x^2 - x - 6)}{x^2 + 3x + 2} \neq f(x) \text{ or } -f(x)$$

It is not even neither odd.

(d) Since $x \neq 2 \Rightarrow f(x) = \frac{2(x+3)(x-2)}{(x-1)(x-2)} = \frac{2(x+3)}{(x-1)}$ for $x \neq 2$.

Now $\lim_{x \to 2} \frac{2(x+3)}{x-1} = \frac{2(2+3)}{2-1} = 10$.

$\Rightarrow x=2$ is not an asymptote

Now

$$\lim_{x \to 1^+} f(x) \overset{x \neq 2}{=} \lim_{x \to 1^+} \frac{2(x+3)}{x-1} = 2(1+3) \lim_{x \to 1^+} \frac{1}{x-1}$$

$$= 8 \lim_{x \to 1^+} \frac{1}{x-1} = +\infty$$

$\Rightarrow \boxed{x=1}$ is a vertical asymptote

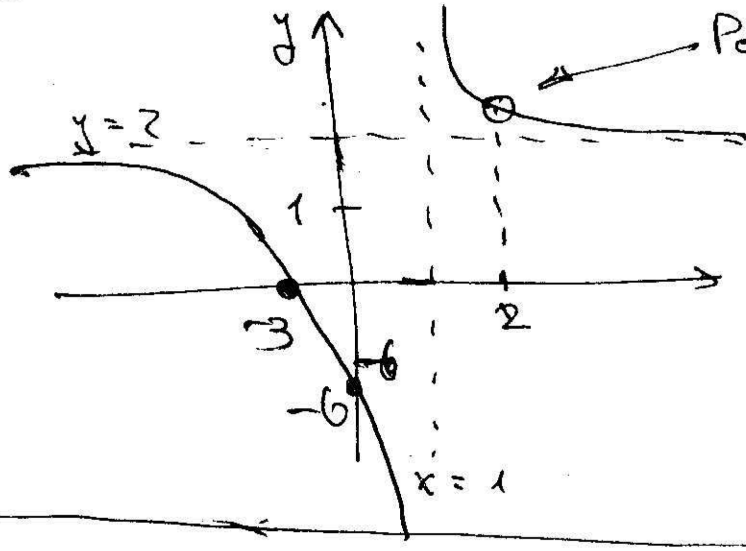
Also

$$\lim_{x \to 1^-} f(x) \overset{x \neq 2}{=} \lim_{x \to 1^-} \frac{2(x+3)}{x-1} = 8 \lim_{x \to 1^-} \frac{1}{x-1} = -\infty.$$

Finally: $\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{2(x+3)}{(x-1)} = \lim_{x \to \pm\infty} \frac{2x(1+\frac{3}{x})}{x(1-1/x)}$

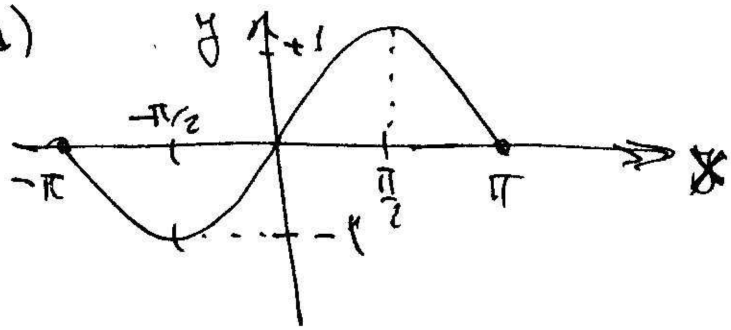
$$= \lim_{x \to \pm\infty} 2\left(\frac{1+3/x}{1-1/x}\right) = 2\left(\frac{1+0}{1-0}\right) = 2 \Rightarrow \boxed{y=2} \text{ Horizontal asymptote}$$

The sketch of the curve is:



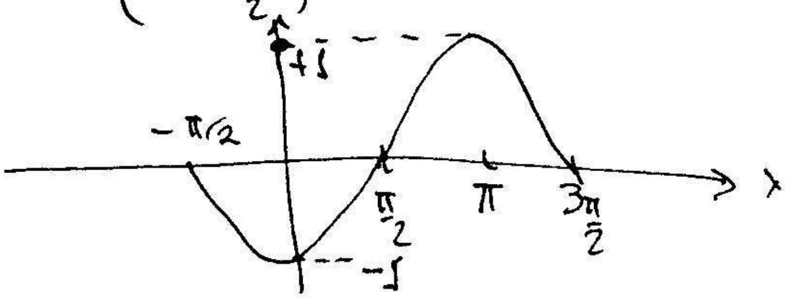
Point (2, 10) is empty!

$\boxed{5}$ (a)

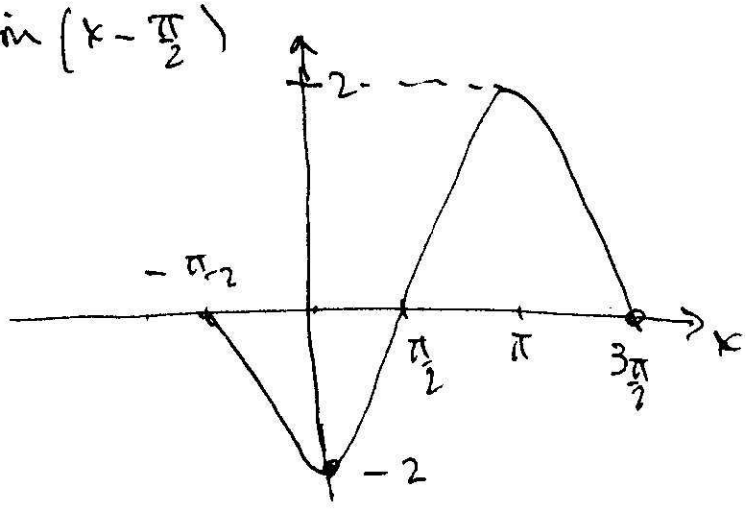


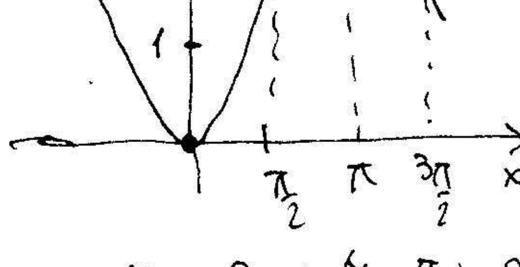
It repeats periodically, with period 2π.

(b) Now: $\sin\left(x - \frac{\pi}{2}\right)$



$2\sin\left(x - \frac{\pi}{2}\right)$





$y = 2\sin\left(x - \frac{\pi}{2}\right) + 2$

(2) Alternative solution

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(-\frac{\pi}{4}\right)$$

$$= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}$$

$$= \frac{1}{4}\left(\sqrt{2} + \sqrt{6}\right)$$

or

$$= \frac{1}{2\sqrt{2}}\left(1 + \sqrt{3}\right)$$

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