Introducción al Cálculo — Viernes, 28 de noviembre de 2016

Universidad Autónoma Metropolitana-Azcapotzalco

Examen #2-B

① Calcule el límite: $\displaystyle\lim_{x\to 0}\frac{\sqrt{x+1}-1}{2-\sqrt{x+4}}$

Substitución directa es indeterminación $\dfrac{\sqrt{0+1}-1}{2-\sqrt{0+4}} = \dfrac{1-1}{2-2} = \text{"}\dfrac{0}{0}\text{"}$

Then, multiply by the conjugate, and by "1":

$$\lim_{x\to 0}\frac{\sqrt{x+1}-1}{2-\sqrt{x+4}}\cdot\frac{(\sqrt{x+1}+1)}{(2+\sqrt{x+4})}\left(\frac{2+\sqrt{x+4}}{\sqrt{x+1}+1}\right) =$$

$$= \lim_{x\to 0}\frac{(x+1)-1}{4-(x+4)}\left(\frac{2+\sqrt{x+4}}{\sqrt{x+1}+1}\right) = \lim_{x\to 0}\frac{x}{-x}\left(\frac{2+\sqrt{x+4}}{\sqrt{x+1}+1}\right)$$

$$= \lim_{x\to 0}(-1)\left(\frac{2+\sqrt{x+4}}{\sqrt{x+1}+1}\right) = (-1)\left(\frac{2+\sqrt{0+4}}{\sqrt{0+1}+1}\right) = (-1)\left(\frac{2+2}{1+1}\right) = \frac{-4}{2} = -2.$$

$$\Rightarrow \boxed{\lim_{x\to 0}\left(\frac{\sqrt{x+1}-1}{2-\sqrt{x+4}}\right) = -2}$$

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② Use: $\cos(2\theta) = 1 - 2\sin^2\theta$.

$$\Rightarrow \quad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Take $\theta = \frac{\pi}{12}$: $\quad \sin^2\left(\frac{\pi}{12}\right) = \dfrac{1-\cos\left(\pi/6\right)}{2} = \dfrac{1-\sqrt{3}/2}{2}$

$$= \frac{2-\sqrt{3}}{4}.$$

$$\Rightarrow \sin\left(\frac{\pi}{12}\right) = \pm\sqrt{\frac{2-\sqrt{3}}{4}}.$$

Since $\frac{\pi}{12}$ is in the first quadrant, $\sin\left(\frac{\pi}{3}\right) > 0$ and choose the "+" sign;

$$\boxed{\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{2-\sqrt{3}}}{2}}$$

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(3) $\lim\limits_{x \to -\infty} \frac{|x+4|}{x+2} = \lim\limits_{x \to -\infty} \frac{|x(1+\frac{4}{x})|}{x(1+2/x)} =$

$= \lim\limits_{x \to -\infty} \frac{|x|\,|1+\frac{4}{x}|}{x(1+2/x)}$

Since $x \to -\infty \Rightarrow x < 0 \Rightarrow |x| = -x$. thus,

$= \lim\limits_{x \to -\infty} \frac{-x\,|1+\frac{4}{x}|}{x(1+2/x)} = \lim\limits_{x \to -\infty} \frac{(-1)|1+\frac{4}{x}|}{(1+2/x)} =$

$= (-1)\frac{|1+0|}{|1+0|} = -1$

$$\Rightarrow \boxed{\lim_{x \to -\infty} \frac{|x+4|}{x+2} = -1}$$

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(4). Bosquejar o gráfico de: $g(x) = 2\frac{(x^2+x-6)}{x^2+x-2}$

(a) Dom(g).

Since $x^2+x-2 = (x+2)(x-1) \Rightarrow g(x) = \frac{2(x-2)(x+3)}{(x+2)(x-1)}$

$\text{Dom}(g) = \mathbb{R} \setminus \{-2, +1\}$

(b) $g(x) = 0 \Rightarrow \frac{2(x-2)(x+3)}{(x+2)(x-1)} = 0 \Rightarrow (x-2)(x+3) = 0$

$\Rightarrow$ X-intersepts. $\boxed{x=-3}$ and $\boxed{x=2}$

and $y = f(0) = \dfrac{2(0+0-6)}{0+0-2} = \dfrac{-12}{-2} = 6.$

$\boxed{y=6}$ is the Y - intersept.

(c) It is not periodic.

Symmitries $f(-x) = \dfrac{2((-x)^2+(-x)-6)}{(-x)^2+(-x)-2} = \dfrac{2(x^2-x-6)}{x^2-x-2} \neq f(x)$ and $\neq -f(x)$

It is not even neither odd.

(d) $\lim\limits_{x\to -2^-} g(x). = \lim\limits_{x\to -2^-} \dfrac{2(x-2)(x+3)}{(x+2)(x-1)} = \dfrac{2(-4)(1)}{(-3)} \lim\limits_{x\to -2^-} \dfrac{1}{x+2}$

$= \dfrac{8}{3} \cdot (-\infty) = -\infty$

$\Rightarrow$ $x=-2$ is a vertical asymptote.

$\lim\limits_{x\to -2^+} g(x) = \lim\limits_{x\to -2^+} \dfrac{2(x-2)(x+3)}{(x+2)(x-1)} = 8 \lim\limits_{x\to -2^+} \dfrac{1}{x+2} = +\infty$

Now:

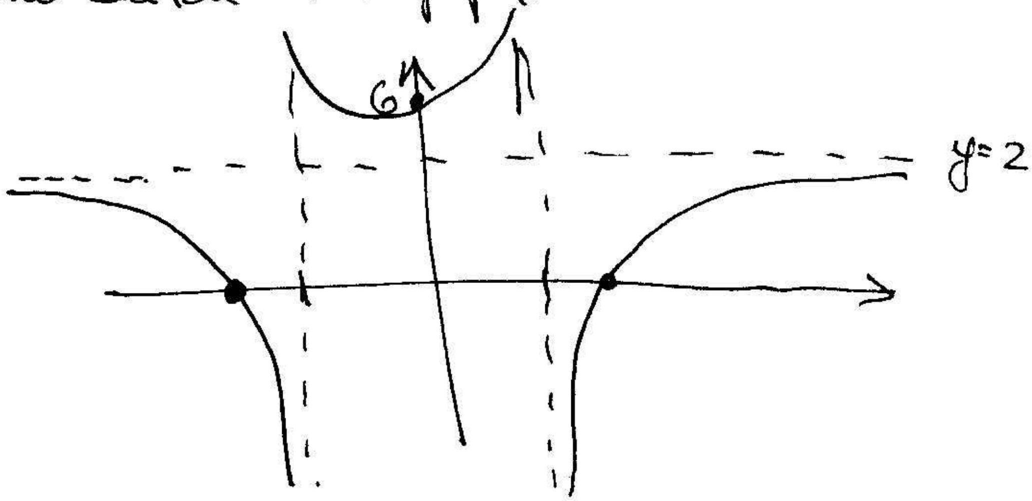
$\lim\limits_{x\to 1^-} g(x) = \lim\limits_{x\to 1^-} \dfrac{2(x-2)(x+3)}{(x+2)(x-1)} = \dfrac{2(-1)4}{3} \lim\limits_{x\to 1^-} \left(\dfrac{1}{x-1}\right)$

$= -\dfrac{8}{3}(-\infty) = \infty$

$\lim\limits_{x\to 1^+} g(x) = \lim\limits_{x\to 1^+} \dfrac{2(x-2)(x+3)}{(x+2)(x-1)} = \dfrac{2(-1)4}{3} \lim\limits_{x\to 1^+} \dfrac{1}{x-1}$

$= -\dfrac{8}{3}(\infty) = -\infty.$

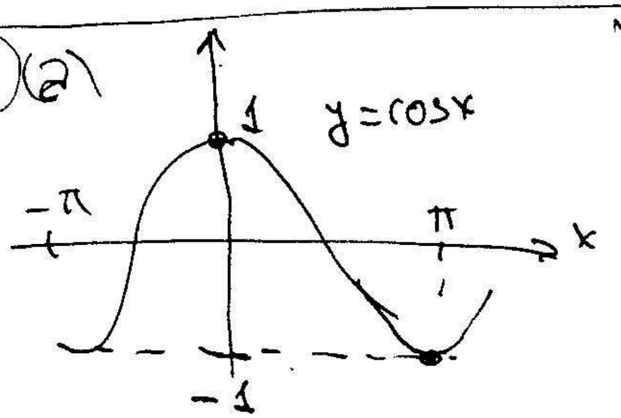
We can now sketch the graph.



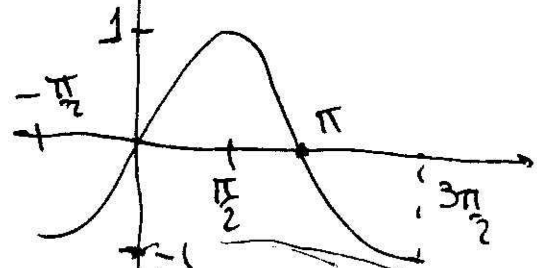
$$\lim_{x\to\infty} \frac{2(x^2+x-6)}{x^2+x-2} = \lim_{x\to\infty} \frac{2(1+1/x-6/x^2)}{1+1/x-2/x^2} = 2$$

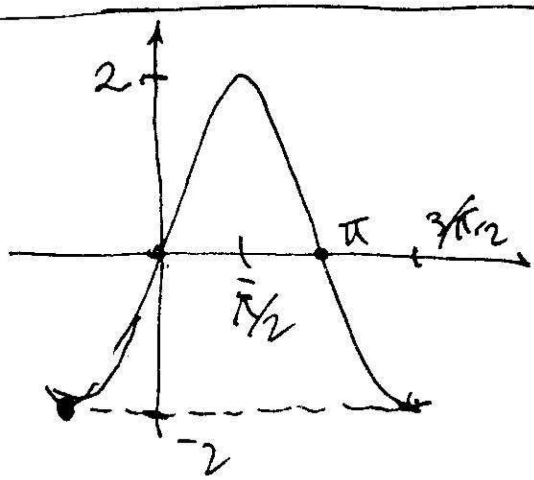
$y = 2$ is the horizontal asymptote.

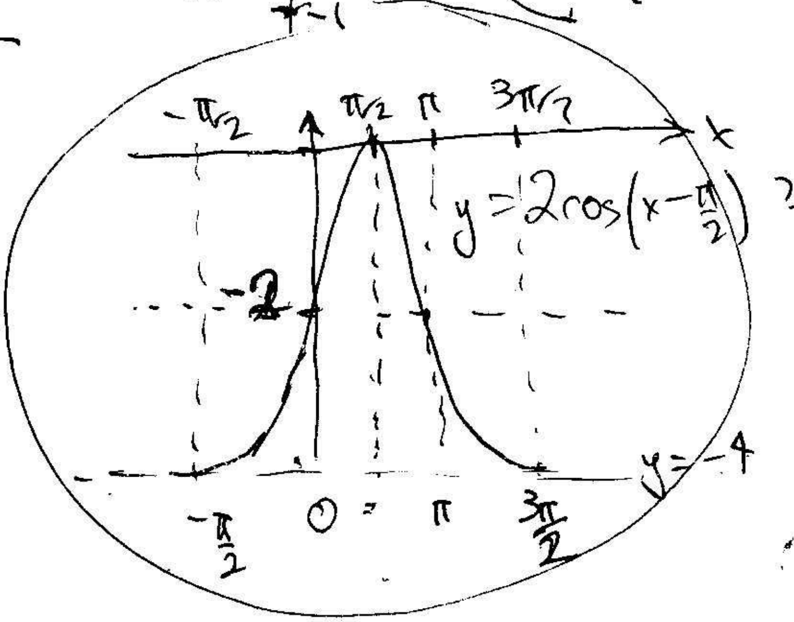
5 (a)



(b) $y = \cos\left(x - \frac{\pi}{2}\right)$







② Alternative solution.

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)$$

$$= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\frac{\pi}{4}\cos\left(\frac{\pi}{3}\right)$$

$$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{2}}{2\cdot 2}\left(\sqrt{3} - 1\right)$$

$$= \frac{1}{4}\left(\sqrt{6} - \sqrt{2}\right)$$

$$\overset{\text{or}}{=} \frac{1}{2\sqrt{2}}\left(\sqrt{3} - 1\right).$$

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$=5 > B$