UAM - Azcapotzalco Ecuaciones Diferenciales Ordinarias

Examen #2 - B

N rems.
17 marzo 2017

(1) $5y'' + 9y' - 2y = 0$

1) Lineal
2) Homogénea
3) Coef. Constantes

$y(t) = e^{rt}$

$5r^2 + 9r - 2 = 0$

~~$r^2 + \frac{9}{5}r - \frac{2}{5} = 0 \Rightarrow (r - \frac{2}{5})(r - \frac{2}{5}) = 0$~~

$$r_{1,2} = \frac{-9 \pm \sqrt{9^2 - 4(5)(-2)}}{2(5)} = \frac{-9 \pm \sqrt{81 + 40}}{10} = \frac{-9 \pm \sqrt{121}}{10}$$

$$= \frac{-9 \pm 11}{10} = \begin{cases} \frac{2}{10} = \frac{1}{5}, \\ -\frac{20}{10} = -2 \end{cases} \quad (r - \frac{1}{5})(r + 2) = 0$$

$$y(t) = C_1 e^{t/5} + C_2 e^{-2t} \qquad \left. \begin{array}{l} C_1 + C_2 = 4 \\ \frac{C_1}{5} - 2C_2 = -2 \end{array} \right\}$$

$$y'(t) = \frac{C_1}{5} e^{t/5} - 2C_2 e^{-2t}$$

$$\left. \begin{array}{l} 2C_1 + 2C_2 = 8 \\ \frac{C_1}{5} - 2C_2 = -2 \end{array} \right\} (2 + \frac{1}{5})C_1 + 0 = 6 \Rightarrow \frac{11}{5} C_1 = 6 \quad \boxed{C_1 = \frac{30}{11}}$$

$$C_2 = 4 - C_1 = 4 - \frac{30}{11} = \frac{44 - 30}{11} = \frac{14}{11} \qquad \boxed{C_2 = \frac{14}{11}}$$

$$\boxed{y(t) = \frac{30}{11} e^{t/5} + \frac{14}{11} e^{-2t}}$$

También:

$$C_2 = 4 - C_1$$

$$\frac{C_1}{5} - 2(4 - C_1) = -2 \qquad \frac{C_1}{5} - 8 + 2C_1 = -2 \qquad (2 + \frac{1}{5})C_1 - 8 = -2$$

$$\frac{11}{5} C_1 = 8 - 2 = 6 \Rightarrow \boxed{C_1 = \frac{30}{11}}$$

$$C_2 = 4 - \frac{30}{11} = \frac{44 - 30}{11} \quad \boxed{C_2 = \frac{14}{11}}$$

$= 1 =$

$$t^2 y'' + 2t\, y' - 6y = 0 \quad , \quad y_1(t) = t^2$$

$$t^2 y_1'' + 2t y_1' - 6y_1 = 2t^2 + 2t(2t) - 6t^2$$

$$= 2t^2 + 4t^2 - 6t^2 = 0 \quad \text{Cumple.}$$

Reducción de orden:

$$y_2(t) = A(t)\, y_1(t).$$

$$t^2 (A'' y_1 + 2A' y_1' + A y_1'') + 2t(A' y_1 + A y_1') - 6(A y_1) = 0$$

Agrupando: $(t^2 y_1) A'' + (2t^2 y_1' + 2t y_1) A' + \underbrace{(t^2 y_1'' + 2t y_1' - 6y_1)}_{=0} A = 0$

Sea $B = A'$. Entonces, ec. 1er orden para B

$$(t^2 y_1) B' + (2t^2 y_1' + 2t y_1) B = 0$$

$$t^4 B' + (4t^3 + 2t^3) B = 0$$

$$t^4 B' + 6t^3 B = 0 \Rightarrow$$

$t \neq 0$:

$$\frac{B'}{B} = -\frac{6}{t} \Rightarrow B(t) = t^{-6} \Rightarrow A(t) = \int B(t) = \int t^{-6}$$

$$\Rightarrow A(t) = \frac{t^{-5}}{-5} \Rightarrow y_2(t) = \frac{t^{-5}}{-5} \cdot t^2. \text{ Solución general}$$

$$\boxed{y(t) = C_1 t^2 + \frac{C_2}{t^3}}$$

(3) Primero resolvemos la ec. homogénea.

$$\frac{d^4 y_h}{dt^4} - y_h = 0$$

1) Lineal
2) Homogénea $y(t) = e^{rt}$.
3) Coef. ctes.

$$\Rightarrow r^4 - 1 = 0 \Rightarrow (r^2-1)(r^2+1) = 0 \quad (r-1)(r+1)(r^2+1) = 0$$

Raíces $r_1 = 1$, $r_2 = -1$, $r_3 = i$, $r_4 = -i$

$$y_h(t) = C_1 e^t + C_2 e^{-t} + C_3 \cos t + C_3 \sin t)$$

Como $g(t) = 3e^t + 5\sin t$.

Primera propuesta: $y_p(t) = Ae^t + B\cos t + C \sin t$

= 2 = No es sensato.

No es sensato porque repite soluciones $y_h(t) \Rightarrow$

Multiplicar por t:

$$\boxed{y_p(t) = Ate^{t} + Bt\cos t + Ct\sin t}$$

(4) Resolver la ec. homogénea: $9y_h'' + y_h = 0$

$$\Rightarrow y_h(t) = C_1\cos\left(\frac{t}{3}\right) + C_2\sin\left(\frac{t}{3}\right).$$

Ahora $y_1 = \cos\left(\frac{t}{3}\right)$

$y_2(t) = \sin\left(\frac{t}{3}\right)$

$$W[y_1, y_2](t) = \det\begin{pmatrix} \cos\left(\frac{t}{3}\right) & \sin\left(\frac{t}{3}\right) \\ -\frac{1}{3}\sin\frac{t}{3} & \frac{1}{3}\cos\left(\frac{t}{3}\right)\end{pmatrix} = \frac{1}{3}$$

$a(t) = 9$

$g(t) = 18\sec^2\left(\frac{t}{3}\right)$

Así:

$$A(t) = -\int \frac{18\sec^2(t/3)\sin(t/3)}{9(1/3)}\,dt = -6\int \frac{\sin(t/3)}{\cos^2(t/3)}\,dt$$

$$= -18\left(\frac{1}{\cos t/3}\right)$$

$$B(t) = \int \frac{18\sec^2(t/3)\cos(t/3)}{9(1/3)}\,dt = 6\int \frac{\cos(t/3)}{\cos^2(t/3)}\,dt = 6\int \sec t/3\,dt$$

$u = t/3$

$$= 18\int \sec u\,du = 18\ln|\sec u + \tan u| = 18\ln\left|\sec\frac{t}{3} + \tan\frac{t}{3}\right|$$

Así:

$$y(t) = C_1\cos\left(\frac{t}{3}\right) + C_2\sin\left(\frac{t}{3}\right)$$

$$+ \left(-\frac{18}{\cancel{\cos t/3}}\right)\cancel{\cos}\left(\frac{t}{3}\right) + \left(18\ln\left|\sec\frac{t}{3} + \tan\frac{t}{3}\right|\right)\sin\frac{t}{3}$$

$$\boxed{y(t) = C_1\cos\left(\frac{t}{3}\right) + C_2\sin\left(\frac{t}{3}\right) - 18 + 18\left(\ln|\sec t + \tan t|\right)\sin t}$$