Fr. Feb 24. 2017

Quiz #5 (Torres HS)-A Ecuaciones Diferenciales Ordinarias

1) $2\dfrac{d^2y}{dt^2} + 6\dfrac{dy}{dt} - 20y = 0.$

$y(0) = 1$

$\dfrac{dy}{dt}(0) = 0$

1) Linear
2) Homogeneous
3) Const. Coeff's

$\Rightarrow y(t) = e^{rt}$

$\Rightarrow 2r^2 + 6r - 20 = 0$

$\Rightarrow r^2 + 3r - 10 = 0 \Rightarrow (r+5)(r-2) = 0 \Rightarrow \boxed{\begin{array}{l} r_1 = -5 \\ r_2 = 2 \end{array}}$

General solution: $\boxed{y(t) = C_1 e^{-5t} + C_2 e^{2t}}$

$y'(t) = -5C_1 e^{-5t} + 2C_2 e^{2t}$

$y(0) = C_1 + C_2 = 1$

$y'(0) = -5C_1 + 2C_2 = 0 \Rightarrow C_2 = \dfrac{5C_1}{2} \Rightarrow C_1 + \dfrac{5}{2}C_1 = 1 \Rightarrow \dfrac{7}{2}C_1 = 1$

$\Rightarrow \boxed{C_1 = \dfrac{2}{7}} \Rightarrow C_2 = \dfrac{5}{2}\cdot\dfrac{2}{7} \Rightarrow \boxed{C_2 = \dfrac{5}{7}}$

$$\boxed{y(t) = \frac{2}{7}e^{-5t} + \frac{5}{7}e^{2t}}$$

---

2) $W[y_1, y_2](t) = \det\begin{pmatrix} t^2 & e^{2(\ln t)} \\ 2t & e^{2\ln t}\left(2\cdot\frac{1}{t}\right) \end{pmatrix} =$

$= t^2\left(e^{2\ln t}\right)\left(2\cdot\dfrac{1}{t}\right) - 2t\left(e^{2\ln t}\right) = t^2\cdot t^2\cdot 2\dfrac{1}{t} - 2t\cdot t^2$

$= 2\dfrac{t^4}{t} - 2t^3 = 2t^3 - 2t^3 = 0.$ Then, the solutions are linearly dependent for all values of t.

Fr. Feb, 24. 2017

Quiz #5-B (Tarea #5) Ecuaciones Diferenciales Ordinarias

(1) $2\frac{d^2y}{dt^2} - 6\frac{dy}{dt} - 20y = 0$

$y(0) = 0$

$\frac{dy}{dt}(0) = 1$

1) Linear
2) Homogeneous
3) Constant Coefficients

$\Rightarrow y(t) = e^{rt}$.

$\Rightarrow 2r^2 - 6r - 20 = 0 \Rightarrow r^2 - 3r - 10 = 0$

$\Rightarrow (r+2)(r-5) = 0 \Rightarrow \boxed{\begin{matrix} r_1 = -2 \\ r_2 = 5 \end{matrix}}$

General solution. $\boxed{y(t) = C_1 e^{-2t} + C_2 e^{5t}}$

$y'(0) = -2C_1 e^{-2t} + 5C_2 e^{5t}$

$y(0) = C_1 + C_2 = 0 \Rightarrow C_2 = -C_1 \Rightarrow -2C_1 - 5C_1 = +1$

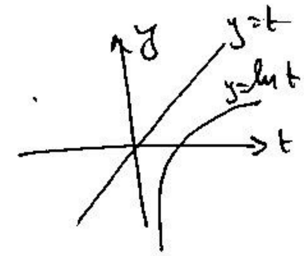
$y'(0) = -2C_1 + 5C_2 = 1 \qquad C_2 = -C_1$

$\Rightarrow -7C_1 = 1 \Rightarrow \boxed{C_1 = -\frac{1}{7}} \quad \boxed{C_2 = \frac{1}{7}} \quad \boxed{y(t) = -\frac{1}{7}e^{-2t} + \frac{1}{7}e^{5t}}$

(2) $W[y_1, y_2](t) = \det\begin{pmatrix} t^2 & \exp(t\ln(t)) \\ 2t & \exp(t\ln t)\left(\ln t + t\cdot\frac{1}{t}\right)\end{pmatrix}$

$= \det\begin{pmatrix} t^2 & t^t \\ 2t & t^t(1+\ln t)\end{pmatrix} = t^{t+2}(1+\ln t) - 2t^{t+1}$

$= t^{t+1}\left[t(1+\ln t) - 2t\right] = t^{t+1}\left[\ln t - t\right]$.



If $t \neq 0 \Rightarrow t^{t+1} \neq 0$. Also $t \neq \ln t$, $\forall t \in \mathbb{R}$

Hence $t^{t+1}[\ln t - t] \neq 0$, if $t \neq 0$.

Hence $y_1(t) = t^2$ and $y_2(t) = \exp(t\ln t) = t^t$ are linearly independent except $t = 0$.