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Quiz #6. Ecuaciones Diferenciales Ordinarias.

SOLUTION KEY

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(1) Solve the Initial Value Problem:

$$9y'' + 6y' + y = 0$$
$$y(0) = 2$$
$$y'(0) = 0$$

The equation is 1) Linear
2) has constant coefficients
and 3) is homogeneous $\Rightarrow y(t) = e^{rt}$

$\Rightarrow$ Characteristic equation $9r^2 + 6r + 1 = 0$

i.e. $r^2 + \frac{6}{9}r + \frac{1}{9} = 0 \Rightarrow r^2 + \frac{2}{3}r + \frac{1}{9} = 0 \Rightarrow \left(r + \frac{1}{3}\right)^2 = 0$

$\Rightarrow r_1 = r_2 = -\frac{1}{3}$ repeated roots.

General solution. $y(t) = (C_1 + C_2 t)e^{-t/3}$

$$y'(t) = \left(C_2 e^{-t/3}\right) + \left(-\frac{1}{3}\right)(C_1 + C_2 t)e^{-t/3}$$
$$= \left(C_2 - \frac{1}{3}C_1 + C_2 t\right)e^{-t/3}$$

Initial conditions imply:

$$C_1 = 2$$
$$C_2 - \frac{1}{3}C_1 = 0 \Rightarrow C_2 = \frac{1}{3}C_1 = \frac{2}{3}$$

$$\Rightarrow \boxed{y(t) = \left(2 + \frac{2}{3}t\right)e^{-t/3}}$$

② Solve the Initial Value Problem:

$$y'' + 4y' + 5y = 0$$
$$y(0) = 1$$
$$y'(0) = 0.$$

The equation is 1) linear
2) homogeneous
and 3) has constant coefficients $\Bigg\}$ $y(t) = e^{rt}$

$r^2 + 4r + 5 = 0 \Rightarrow$ Chicharronera $r_{1,2} = \dfrac{-4 \pm \sqrt{4^2 - 4 \cdot 5}}{2}$

$$r_{1,2} = \frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = -2 \pm i.$$

$$\Rightarrow \quad y(t) = e^{-2t} e^{\pm it} = e^{-2t}(\cos t \pm i \sin t)$$

General Real solution:

$$y(t) = (A \cos t + B \sin t)\, e^{-2t}$$

$$\dot{y}(t) = (-A \sin t + B \cos t)\, e^{-2t} - 2(A \cos 2t + B \sin 2t)\, e^{-2t}$$

Initial Conditions:

$$(A + 0) \cdot 1 = 1 \Rightarrow \boxed{A = 1}$$

$$(0 + B) \cdot 1 - 2 \cdot (A + 0) = 0 \Rightarrow \quad B = 2A \Rightarrow \boxed{B = 2}$$

$$\boxed{y(t) = (\cos t + 2 \sin t)\, e^{-2t}}$$