UAM- Azcapotzalco Viernes, Marzo 10, 2017

Ecuaciones Diferenciales Ordinarias Quiz #7

ANSWER KEY

(1) Since $y_1(t) = t^{-1}$ is a solution to the differential equation,

(*) $$t^2 y'' + 3t y' + y = 0, \quad \text{with domain } t > 0, \ (*)$$

find a second, linearly independent, solution to the equation.

Use the method of reduction of order (I always forget the formula). Thus:
$$y_2(t) = v(t)\, y_1(t) = v(t)\, t^{-1},$$

We must find $v(t)$:
$$y_2' = v' t^{-1} - v t^{-2}$$
$$y_2'' = v'' t^{-1} - 2v' t^{-2} + 2v t^{-3}$$

thus, eq'n (*) becomes:
$$t^2\left(v'' t^{-1} - 2v' t^{-2} + 2v t^{-3}\right) + 3t\left(v' t^{-1} - v t^{-2}\right) + v t^{-1} = 0$$

i.e. $$v''\left(\frac{t^2}{t}\right) + v'\left(\frac{-2t^2}{t^2} + 3\right) + v\left(\frac{2 t^2}{t^3} - \frac{3t}{t^2} + \frac{1}{t}\right) = 0$$

i.e. $$t v'' + v' = 0.$$

Reduction of order: $V = v' \Rightarrow tV' + V = 0$

$$\Rightarrow \frac{V'}{V} = -\frac{1}{t} \Rightarrow \frac{d}{dt}\operatorname{Log} V = -\frac{d}{dt}\operatorname{Log} t \Rightarrow \operatorname{Log} V = -\operatorname{Log} t.$$

$$\Rightarrow V = t^{-1} \Rightarrow v' = t^{-1} \Rightarrow \boxed{v(t) = \operatorname{Log} t}$$

Therefore, $y_2(t) = (\operatorname{Log} t)\, t^{-1} \Rightarrow \boxed{y_2(t) = \frac{\operatorname{Log} t}{t}}$

(2) Find a particular solution to:

$$y'' + 2y' + 2y = 2e^{-t}\cos t.$$

To find a particular solution, we require first to find a solution to the homogeneous equation:

$$y_h'' + 2y_h' + 2y_h = 0$$

which has a characteristic eq'n:

$$r^2 + 2r + 2r = 0$$

By the Chicharronera's formula

$$r = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2} = \frac{-2 \pm \sqrt{4}\sqrt{1-2}}{2}.$$

i.e. $r_{1,2} = -1 \pm i.$

Hence:

$$y_h(t) = e^{(-1+i)t} = e^{-t}e^{it}$$
$$= e^{-t}(\cos t + i \sin t)$$

The real sol'n:

$$y_h(t) = C_1 e^{-t}\cos t + C_2 e^{-t}\sin t.$$

For the particular solution, the first trial is:

$g(t) = 2e^{-t}\cos t \Rightarrow y_{p,1}(t) = Ae^{-t}\cos t + e^{-t}B\sin t,$ ← NO!

But this is repeated: Multiply by t:

$$\boxed{y_p(t) = te^{-t}A\cos t + te^{-t}B\sin t}$$