1.a.✓ $f'(x) = \cancel{\quad} = \dfrac{x(\sec x \tan x) + \sec x}{2\sqrt{7 + x\sec x}}$

1.b.✓ $g'(x) = \dfrac{(x+7)^4 \sec^2(3x)(3) - (\tan 3x)(4(x+7)^3)}{(x+7)^8}$

1.c.✓ $h'(x) = 3\left(1+\tan^4\left(\frac{x}{12}\right)\right)^2\left(4\tan^3\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{12}\right)\left(\frac{1}{12}\right)\right)$

2.a.✓ $0 = 2\sin(\pi)$

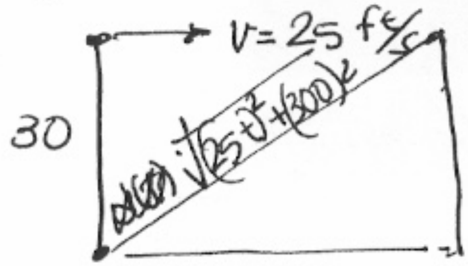
$\frac{dy}{dx} = 2\cos(\pi x - y)\left[\pi - \frac{dy}{dx}\right] \therefore \frac{dy}{dx}\left[1 + 2\cos(\pi x - y)\right] = 2\pi\cos(\pi x - y)$

$\therefore \frac{dy}{dx} = \frac{2\pi\cos(\pi x - y)}{1 + 2\cos(\pi x - y)}\Big|_{(1,0)} = \frac{-2\pi}{1-2} = 2\pi \quad \therefore y = 2\pi(x-1)$

3/3

3.V

altura = 300 constante.



$\Rightarrow d(t) = 25t$

$d(t) = 500 \Rightarrow t = \frac{500}{25} = 20$

$$D(t) = \sqrt{(25t)^2 + 300^2} = \sqrt{625t^2 + 300^2}$$

$$D'(t) = \frac{\cancel{2}(625)\,t}{\cancel{2}\sqrt{625t^2 + 300^2}}\Bigg|_{t=20} = \frac{12500}{\sqrt{(625)(400) + 300^2}}$$

$$= \frac{12500}{\sqrt{2500 + 300^2}} = \frac{12500}{\sqrt{50^2 + 6^2 50^2}} = \frac{1250\cancel{0}}{5\cancel{0}\sqrt{37}} = \frac{250}{\sqrt{37}}$$

CÁLCULO DIFERENCIAL Parte II. Vespertino - 17P

(1) $\frac{dg}{dx} = -2\sin x(\cos x + 1)$, $x \in [0, 2\pi]$.

1.(a) Puntos críticos: (i) $x=0$, $x=2\pi$: puntos frontera

(ii) $g'(x)$ siempre existe.

(iii) $g'(x)=0$ en $x = 0, \pi, \frac{3\pi}{2}, 2\pi$.

Si $g'(x)=0$ entonces: $\sin x = 0$ ó $\cos x = -1$.

- $\sin x = 0$ en $x = 0, \pi, 2\pi$.
- $\cos x = -1$ en $x = \frac{3\pi}{2}$

Pts críticos: $x_1 = 0$, $x_2 = \pi$, $x_3 = \frac{3}{2}\pi$, $x_4 = 2\pi$.

1(b) $x \in (0,\pi)$: $\sin x > 0$, $1+\cos x > 0 \Rightarrow g' < 0$, g decreciente

$x \in (\pi, \frac{3\pi}{2})$ $\sin x < 0$, $1+\cos x > 0 \Rightarrow g' > 0$ g creciente

$x \in (\frac{3}{2}\pi, 2\pi)$ $\sin x < 0$, $1+\cos x > 0 \Rightarrow g' > 0$ g decreciente.

Entonces:

g decreciente en $(0,\pi)$

g creciente en $(\pi, \frac{3}{2}\pi)$

g creciente en $(\frac{3}{2}\pi, 2\pi)$.

1(c) $g(0)$ máximo local.

$g(\pi)$ mínimo local.

$g(2\pi)$ máximo local

② Sea $f(x) = \dfrac{x^2-49}{x^2+5x-14}$.

Notemos que: $f(x) = \dfrac{(x-7)(x+7)}{(x+7)(x-2)}$

(a) $\text{Dom}(f) = \mathbb{R} \setminus \{-7, 2\}$

Podemos simplificar: $f(x) = \dfrac{x-7}{x-2}$; $x \neq 2, x \neq -7$

$f(x) = 0$, si $x = 7$, solamente

(b) Dado que:

$$\left.\begin{array}{l} \lim\limits_{x \to 2^{\pm}} f(x) = \mp \infty \\ \lim\limits_{x \to \pm\infty} f(x) = +1 \end{array}\right\}$$

entonces: $x = 2$ es asíntota vertical

mientras que: $y = 1$ asíntota horizontal

(c) Debe calcular $f'(x)$:

Notar que: $f(x) = 1 - \dfrac{5}{x-2}$

Entonces: $f'(x) = \dfrac{5}{(x-2)^2}$

Pts críticos: (i) No hay fronteras.

(ii) $f'(x)$ no existe en $x = 2$, $x = -7$.

(iii) $f'(x) = 0$ nunca, i.e., $f'(x) \neq 0$.

Puntos críticos: $x_2 = 2$, $x_1 = -7$.

(d) Interválos de monotonía:

Note that $f'(x) = \frac{5}{(x-2)^2} > 0, \quad x \neq 2, \; x \neq -7.$

Then, f is creciente en $(-\infty,-7)$, $(-7,2)$ y $(2,\infty)$.

(e) Interválos de concavidad.

$$f''(x) = -\frac{10}{(x-2)^3}$$

If $x \in (-\infty,-7) \cup (-7,2)$, $f'' > 0$

f cóncava hacia arriba

If $x \in (2,\infty)$, $f'' < 0$

f cóncava hacia abajo.

$x = 2$ no es punto de inflexión pues $x \notin Dom(f)$.

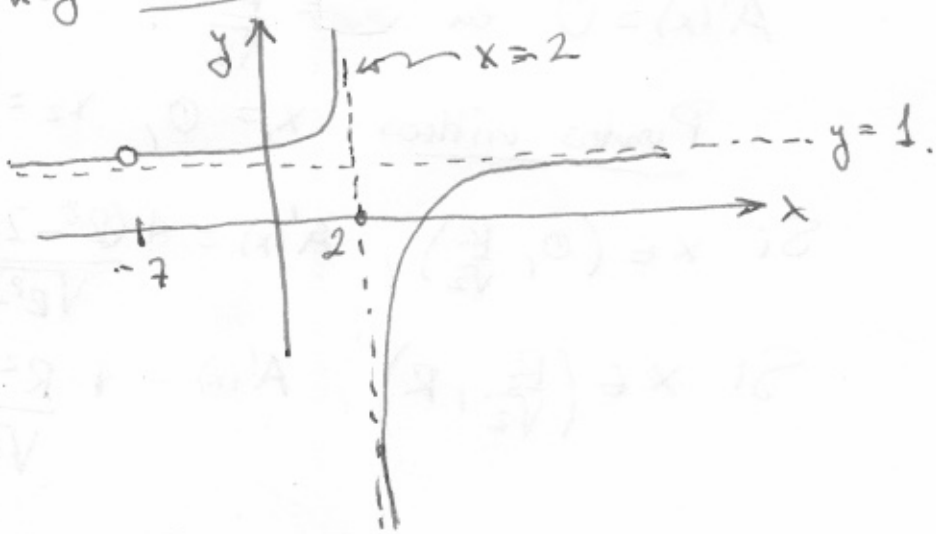
(f) Como tiene asíntotas verticales y horizontales:

$$\lim_{x \to 2^{\pm}} f(x) = \mp \infty .$$

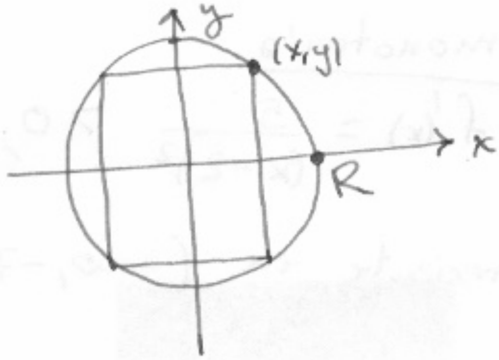
no hay extremos absolutos.

Tampoco hay extremos locales.

(g) Gráfica:



(3)



$R = 10$ cm.

$\text{Area} = (2x)(2y) = 4xy.$

Restricción: $x^2+y^2=R^2 \Rightarrow y=\sqrt{R^2-x^2}$, $x\in[0,R]$

$A(x) = 4x\sqrt{R^2-x^2}$

$\text{Dom}(A) = [0,R]$, $A(0) = 0$

$A(R) = 0.$

Pts críticos: (a) Frontera $x=0$, $x=R$

(b) $A'(x)$ no existe en $x=+R$

(c) $A'(x)=0$ en $x=R/\sqrt{2}$

$$A'(x) = 4\sqrt{R^2-x^2} + 4x\frac{(-2x)}{2\sqrt{R^2-x^2}} = 4\left(\frac{(R^2-x^2)-x^2}{\sqrt{R^2-x^2}}\right)$$

$$= 4\left(\frac{R^2-2x^2}{\sqrt{R^2-x^2}}\right).$$

$A'(x)=0$ en $x=\frac{R}{\sqrt{2}}$.

Puntos críticos $x_1=0$, $x_2=\frac{R}{\sqrt{2}}$, $x_3=R$.

Si $x\in(0,\frac{R}{\sqrt{2}})$, $A'(x)=4\frac{(R^2-2x^2)}{\sqrt{R^2-x^2}}>0$, A creciente

Si $x\in(\frac{R}{\sqrt{2}},R)$, $A'(x)=4\frac{R^2-2x^2}{\sqrt{R^2-x^2}}<0$, A decreciente

Entonces $A\left(\frac{R}{\sqrt{2}}\right)$ es máximo absoluto

$$A\left(\frac{R}{\sqrt{2}}\right) = 4\,\frac{R}{\sqrt{2}}\sqrt{R^2-\frac{R^2}{2}} = 4\,\frac{R}{\sqrt{2}}\sqrt{\frac{R^2}{2}} = 4\,\frac{R}{\sqrt{2}}\,\frac{R}{\sqrt{2}}$$

$$\boxed{A\left(\frac{R}{\sqrt{2}}\right) = 2R^2}$$

$$x = \frac{R}{\sqrt{2}}\ ;\ y = \sqrt{R^2-x^2} = \sqrt{R^2-\frac{R^2}{2}} = \sqrt{\frac{R^2}{2}} = \frac{R}{\sqrt{2}}.$$

$$\left.\begin{array}{l} \text{Base} = 2x = \frac{2}{\sqrt{2}}R = \sqrt{2}R \\ \text{Altura} = 2y = \frac{2R}{\sqrt{2}} = \sqrt{2}R \end{array}\right\}\ \text{El rectángulo es cuadrado}$$

Cálculo Diferencial $\underline{III}$ Parte Vespertino 17P

1.a) $y = (\operatorname{arc\,sen}(3x^2))^{\cos x}$

$\ln y = \cos x \ln(\operatorname{arc\,sen}(3x^2))$ derivando

$$\frac{y'}{y} = \cos x \;\frac{1}{\operatorname{arc\,sen}(3x^2)}\;\frac{6x}{\sqrt{1-(3x^2)^2}} + \ln(\operatorname{arc\,sen}(3x^2)\cdot(-\operatorname{sen}x)$$

$$y' = (\operatorname{arc\,sen}(3x^2))^{\cos x}\left[\frac{6x\cos x}{\operatorname{arc\,sen}(3x^2)\sqrt{1-9x^4}} - \operatorname{sen}x \ln(\operatorname{arc\,sen}3x^2)\right]$$

1b) $y = \ln\frac{\sqrt{2x^3-1}}{4x^2+7} = \frac{1}{2}\ln(2x^3-1) - \ln(4x^2+7)$

$$y' = \frac{6x^2}{2(2x^3-1)} - \frac{8x}{(4x^2+7)} = \frac{3x^2}{(2x^3-1)} - \frac{8x}{4x^2+7}$$

1c) $y = \cos(e^{x^4-x^2})$

$$y' = -\operatorname{sen}(e^{x^4-x^2})\cdot e^{x^4-x^2}(4x^3-2x) = -(4x^3-2x)e^{x^4-x^2}\operatorname{sen}(e^{x^4-x^2})$$

2)

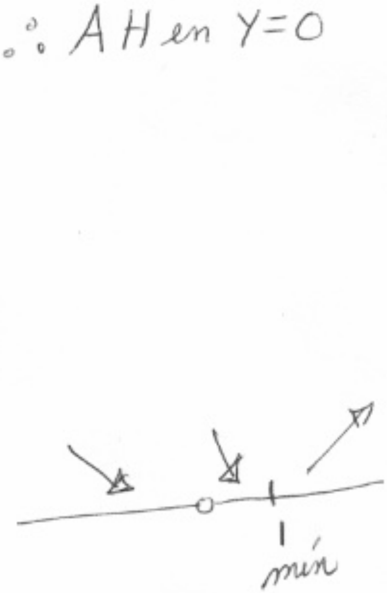
a) Dom $\mathbb{R}-\{0\}$, ceros no tiene

b) $\lim\limits_{x\to-\infty}\frac{e^x}{3x} = \lim\limits_{x\to-\infty} e^x\cdot\frac{1}{3x} = 0\cdot 0^- = 0^-$ $\therefore$ AH en $y=0$

$$\lim_{x\to\infty}\frac{e^x}{3x} \overset{\text{L'Hôpital}}{=} \lim_{x\to\infty}\frac{e^x}{3} = \infty$$

$$\left.\begin{array}{l}\lim\limits_{x\to 0^-}\frac{e^x}{3x} = \frac{1}{0^-} = -\infty \\ \lim\limits_{x\to 0^+}\frac{e^x}{3x} = \frac{1}{0^+} = \infty\end{array}\right\}\therefore \text{ AV en } x=0$$

c) $f' = \frac{3xe^x - 3e^x}{9x^2} = \frac{e^x(x-1)3}{9x^2}$ PC $\{1\}$



mínimo $(1, \frac{e}{3})$

3) $f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f''(x) = \sec^3 x + \tan^2 x \sec x$

$f'''(x) = 3\sec^2 x \sec x \tan x + \tan^2 x \sec x \tan x + \sec x \, 2\tan x \sec^2 x$

$= 5\sec^3 x \tan x + \tan^3 x \sec x$

$f(\pi/6) = \sec(\pi/6) = \frac{2}{\sqrt{3}}$

$f'(\pi/6) = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{3}} = \frac{2}{3}$

$f''(\pi/6) = \left(\frac{2}{\sqrt{3}}\right)^3 + \left(\frac{1}{\sqrt{3}}\right)^2 \frac{2}{\sqrt{3}} = \frac{8}{3\sqrt{3}} + \frac{2}{3\sqrt{3}} = \frac{10}{3\sqrt{3}}$

$f'''(\pi/6) = 5\left(\frac{2}{\sqrt{3}}\right)^3 \frac{1}{\sqrt{3}} + \left(\frac{1}{\sqrt{3}}\right)^3 \frac{2}{\sqrt{3}} = \frac{8 \cdot 5}{9} + \frac{2}{9} = \frac{42}{9}$

$P_3(x) = \frac{2}{\sqrt{3}} + \frac{2}{3}(x - \pi/6) + \frac{10}{3\sqrt{3} \cdot 2}(x - \pi/6)^2 + \frac{42}{9 \cdot 6}(x - \pi/6)^3$

$P_3(x) = \frac{2}{\sqrt{3}} + \frac{2}{3}(x - \pi/6) + \frac{5}{3\sqrt{3}}(x - \pi/6)^2 + \frac{7}{9}(x - \pi/6)^3$

$P_3\left(\frac{7\pi}{45}\right) = \frac{2}{\sqrt{3}} + \frac{2}{3}\left(-\frac{\pi}{90}\right) + \frac{5}{3\sqrt{3}}\left(-\frac{\pi}{90}\right)^2 + \frac{7}{9}\left(-\frac{\pi}{90}\right)^3 = 1.2325688737$

$\frac{2}{\sqrt{3}} - \frac{\pi}{135} + \frac{\pi^2}{4860\sqrt{3}} - \frac{7\pi^3}{6561000}$

4) $g(x) = e^{-3\ln x^2} = e^{\ln (x^2)^{-3}} = e^{\ln x^{-6}} = x^{-6} = \frac{1}{x^6}$

a) Dom g: $\mathbb{R} - \{0\}$

b) $g' = -6x^{-7}$ g crece en $(-\infty, 0)$ aquí existe la inversa

$= -\frac{6}{x^7}$ g decrece en $(0, \infty)$

c) $\frac{dg^{-1}}{dy}(e^6) = \frac{1}{g'(\frac{1}{e})} = \frac{1}{-6e^7}$

$g' = -\frac{6}{x^7}$

$g'(\frac{1}{e}) = -\frac{6}{\frac{1}{e^7}}$

$= -6e^7$

d) $y = \frac{1}{x^6}$ $x^6 = \frac{1}{y}$ $x = \frac{1}{\sqrt[6]{y}}$

$g^{-1}(y) = \frac{1}{\sqrt[6]{y}}$ $y > 0$ para $x \in (0, \infty)$