Universidad Autónoma Metropolitana Azcapotzalco. | Vi, 19 mayo 2017

CÁLCULO DIFERENCIAL.

Cuestionario #1

Nombre: ANSWER KEY

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1. Usando la definición de derivada, calcule la derivada de la función

$$f(x) = \frac{1}{x}.$$

2. Usando la linealización de $f(x) = \sqrt{x}$, aproxime $\sqrt{15}$.

(1.) We need to compute:

(a) $f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x} = \frac{x-(x+h)}{(x+h)x} = \frac{-h}{(x+h)x}$

(b) $\frac{f(x+h) - f(x)}{h} = \frac{1}{h}\,\frac{-h}{(x+h)x} = \frac{-1}{(x+h)x}.$

(c) $\frac{df}{dx}(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{-1}{(x+h)x} = -\frac{1}{x^2}$

Hence: $\frac{df}{dx} = \frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

(2.) The linearization is the linear function given by the equation of the tangent line at $x = a$; $y = y_0$:

$$y = m(x-a) + y_0.$$

In this instance: $m = f'(a)$, $y_0 = f(a)$. Hence,

$$L(x) = f'(a)(x-a) + f(a)$$

Choose $a = 16$, then: $f(a) = \sqrt{a} = \sqrt{16} = 4$.

since $16 \sim$ $= 1 =$

Now $f'(x) = \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{\frac{1}{2}-1}$

$= \frac{1}{2}x^{-1/2} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}}.$

Then $f'(a) = f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{8}$

Hence, the linearization is:

$$L(x) = 4 + \frac{1}{8}(x-16).$$

We approximate $\sqrt{16}$ by $L(15)$

$f(15) = \sqrt{15} \approx L(15) = 4 + \frac{1}{8}(15-16) = 4 - \frac{1}{8} = \frac{32-1}{8}$

$\Rightarrow \boxed{\sqrt{15} \approx \frac{31}{8}}$ or $\sqrt{15} \approx 4 - \frac{1}{8} = 4 - 0.125$

or $\boxed{\sqrt{15} \approx 3.875}$

No calculator needed.
