Universidad Autónoma Metropolitana - Azcapotzalco

CÁLCULO DIFERENCIAL A Cuestionario #2 (Viernes 26 de mayo de 2017)

Nombre: ANSWER KEY. ANSWER KEY.

1. Calcule la derivada de:

$$f(x) = \sin\left(\frac{x}{1+x^2}\right) + \cos\left(\frac{x}{1+x^2}\right)$$

2. Determine $\frac{dF}{d\theta}$

$$F(\theta) = \sec\theta \csc\theta.$$

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1. Let $u \equiv \frac{x}{1+x^2}$. Notice $u = u(x)$ then.

$$f'(x) = \frac{d}{dx}\left(\sin u + \cos u\right) = \frac{d}{du}\left(\sin u + \cos u\right)\frac{du}{dx}$$ By the, Chain rule

$$= \left(\cos u - \sin u\right)\frac{d}{dx}\left(\frac{x}{1+x^2}\right)$$

$$= \left(\cos u - \sin u\right)\frac{(1+x^2)(x)' - x(1+x^2)'}{(1+x^2)^2}$$, by quotient rule

$$= \left(\cos u - \sin u\right)\frac{(1+x^2) - x(2x)}{1+x^2}$$

is.

$$\boxed{f'(x) = \left(\cos\left(\frac{x}{1+x^2}\right) - \sin\left(\frac{x}{1+x^2}\right)\right)\frac{1-x^2}{1+x^2}}$$

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2. $$\frac{dF}{d\theta} = \frac{d}{d\theta}\left(\frac{1}{\cos\theta \sin\theta}\right) = -\frac{1}{(\cos\theta \sin\theta)^2}\frac{d}{d\theta}(\cos\theta \sin\theta)$$ by Chain rule

$$= -\frac{(-\sin^2\theta + \cos^2\theta)}{\cos^2\theta \sin^2\theta} = \boxed{\frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta}} = \boxed{\sec^2\theta - \csc^2\theta}$$

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A different form of solution;

$$\frac{dF}{d\theta} = \frac{d}{d\theta}\left(\sec\theta \csc\theta\right) = \frac{d}{d\theta}(\sec\theta)\csc\theta + \sec\theta \frac{d}{d\theta}(\csc\theta)$$

$$= \frac{d}{d\theta}\left(\frac{1}{\cos\theta}\right)\csc\theta + \sec\theta \frac{d}{d\theta}\left(\frac{1}{\sin\theta}\right)$$

$$= -\frac{1}{\cos^2\theta}(-\sin\theta)\csc\theta + \sec\theta\left(\frac{-1}{\sin^2\theta}\right)\cos\theta$$

$$= +\sec\theta \underbrace{\tan\theta \csc\theta} - \sec\theta \csc\theta \cot\theta$$

$$= \sec\theta \frac{\sin\theta}{\cos\theta}\frac{1}{\sin\theta} - \frac{1}{\cos\theta}\csc\theta \frac{\cos\theta}{\sin\theta}$$

$$= \sec\theta \frac{1}{\cos\theta} - \csc\theta \cdot \frac{1}{\sin\theta}$$

$$= \frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta}$$

$$= \sec^2\theta - \csc^2\theta$$ Same result b

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CÁLCULO DIFERENCIAL B Cuestionario #2. Viernes 26 de mayo 2017

Nombre: ANSWER KEY.

1 Calcule la derivada de

$$g(x) = \cos\left(\frac{x^2}{1+x}\right) + \tan\left(\frac{x^2}{1+x}\right).$$

2. Determine $\frac{dG}{d\theta}$.

$$G(\theta) = \theta \sin\theta + \cos\theta$$

1. Let $u = \frac{x^2}{1+x}$

$$\frac{dg}{dx} = \frac{d}{dx}(\cos u + \tan u) = \frac{d}{du}(\cos u + \tan u)\frac{du}{dx}$$ Chain rule

$$= \left(-\sin(u) + 1 + \tan^2 u\right)\frac{d}{dx}\left(\frac{x^2}{1+x}\right)$$

$$= \left(-\sin u + 1 + \tan^2 u\right)\frac{(1+x)(x^2)' - x^2(1+x)'}{(1+x)^2},$$ Quotient rule.

$$= \left(-\sin u + 1 + \tan^2 u\right)\frac{(1+x)2x - x^2\cdot 1}{(1+x)^2}$$

$$\boxed{\frac{dg}{dx} = \left(-\sin\left(\frac{x^2}{1+x}\right) + 1 + \tan^2\left(\frac{x^2}{1+x}\right)\right)\frac{2x+x^2}{(1+x)^2}}$$

or

$$\boxed{\frac{d}{dx}g = \left(-\sin\left(\frac{x^2}{1+x}\right) + \sec^2\left(\frac{x^2}{1+x}\right)\right)\frac{2x+x^2}{(1+x)^2}}$$

2. $\frac{dG}{d\theta} = \frac{d}{d\theta}\left(\theta \sin\theta + \cos\theta\right)$

$$= \frac{d\theta}{d\theta}\sin\theta + \theta\frac{d\sin\theta}{d\theta} + \frac{d\cos\theta}{d\theta}$$

$$= 1 \cdot \sin\theta + \theta\cos\theta - \sin\theta$$

$$\boxed{\frac{dG}{d\theta} = \theta\cos\theta}$$

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CÁLCULO DIFERENCIAL C. Cuestionario #2 | Viernes 26 de mayo de 201

Nombre: KEY.

1. Calcule la derivada de:

$$h(x) = \tan\left(\frac{x}{1+x}\right) + \sin\left(\frac{x}{1+x}\right)$$

2. Determine $\frac{dH}{d\theta}$:

$$H(\theta) = (1+\sec\theta)\sin\theta.$$

1. Let $u = \frac{x}{1+x}$.

$$\frac{dh}{dx} = \frac{d}{dx}(\tan u + \sin u) = \frac{d}{du}(\tan u + \sin u)\frac{du}{dx},$$ Chain rule

$$= ((1+\tan^2 u) + \cos u)\frac{d}{dx}\left(\frac{x}{1+x}\right)$$

$$= ((1+\tan^2 u) + \cos u)\frac{(1+x)(x)' - x(1+x)'}{(1+x)^2}$$ Quotient rule.

$$= (1+\tan^2 u + \cos u)\frac{(1+x)\cdot 1 - x\cdot 1}{(1+x)^2}$$

$$\boxed{\frac{dh}{dx} = \left(1+\tan^2\left(\frac{x}{1+x}\right) + \cos\left(\frac{x}{1+x}\right)\right)\frac{1}{(1+x)^2}}$$

or

$$\boxed{\frac{dh}{dx} = \left(\sec^2\left(\frac{x}{1+x}\right) + \cos\left(\frac{x}{1+x}\right)\right)\frac{1}{(1+x)^2}}$$

2: $\frac{d}{d\theta}H = \frac{d}{d\theta}\Big((1+\sec\theta)\sin\theta\Big)$

$$= \frac{d}{d\theta}(1+\sec\theta)\sin\theta + (1+\sec\theta)\frac{d}{d\theta}\sin\theta$$

$$= \left(0 + \frac{d\sec\theta}{d\theta}\right)\sin\theta + (1+\sec\theta)\cos\theta$$

$$= \frac{d}{d\theta}\left(\frac{1}{\cos\theta}\right)\sin\theta + (\cos\theta + \sec\theta\cos\theta)$$

$$= -\frac{1}{\cos^2\theta}\left(\frac{d\cos\theta}{d\theta}\right)\sin\theta + (\cos\theta + 1)$$

$$= -\frac{1}{\cos^2\theta}(-\sin\theta)\sin\theta + (\cos\theta + 1)$$

$$= \frac{\sin^2\theta}{\cos^2\theta} + (1+\cos\theta)$$

$$\boxed{\frac{dH}{d\theta} = \tan^2\theta + 1 + \cos\theta.}$$

or $\boxed{\frac{dH}{d\theta} = \sec^2\theta + \cos\theta}$