Quiz #7: Examen tipo A. CÁLCULO DIFERENCIAL | Viernes 30 de junio 2017

(1.) Graficar $f(x) = x^2 - \frac{2}{x}$.

1. Dominio y simetrías de $f(x)$. No es par ni impar.

$\mathrm{Dom}(f) = (-\infty, 0) \cup (0, \infty)$ o bien $\mathbb{R} \setminus \{0\}$.

2. Derivadas:

$$f'(x) = 2x + \frac{2}{x^2} = 2x\left(1 + \frac{1}{x^3}\right)$$

$$f''(x) = 2 - \frac{4}{x^3} = 2\left(1 - \frac{2}{x^3}\right)$$

3. Puntos críticos (a) No hay puntos frontera.

(b) $f'(x)$ no existe en $x=0$ (pero no está en $\mathrm{Dom}(f)$)

(c) $f'(x)=0$.

$f'(x) = 0$, i.e., $2x + \frac{2}{x^2} = 0$, i.e. $x^3 = -1$

i.e., $\boxed{x_1 = -1}$ es el único punto crítico.

4. Comportamiento: crecimiento y decrecimiento.

$f'(x) > 0$, i.e. $2x\left(1 + \frac{1}{x^3}\right) > 0$.

If $x > 0$, $1 + \frac{1}{x^3} > 0$, and this holds, for all $x > 0$.

then $f \nearrow$, in $(0, \infty)$.

If $x < 0$, $\left(1 + \frac{1}{x^3}\right) < 0$, $1 < -\frac{1}{x^3}$, $x^3 < -1$,

i.e., $x < -1$, then, $f \searrow$ for $(-\infty, -1)$

If $-1 < x < 0$, $f \nearrow$ for $(-1, 0)$.

$\boxed{-1 = }$

Then, $x_1 = -1$ is a <u>local</u> <u>minimum</u>.

5. <u>Concavidad</u> y <u>pts. de inflexión</u>.

- $f''(x) = 2\left(1 - \frac{2}{x^3}\right) > 0$, i.e., $1 - \frac{2}{x^3} > 0$, $1 > \frac{2}{x^3}$.

If $x > 0$: $x^3 > 2 \Rightarrow$, $x > \sqrt[3]{2}$.

then f is upward concave, if $\underline{x > \sqrt[3]{2}}$.

If $x < 0$, $\frac{-2}{(-x^3)} > -1$, $2 > x^3$, $\sqrt[3]{2} > x$:

then, $x < 0$ and $x < \sqrt[3]{2} \Rightarrow \boxed{x < 0}$

If $x < 0$, f is <u>upwards</u> <u>concave</u>. This confirms $f(x_1)$ is loc. min.

- If $f''(x) = 2\left(1 - \frac{2}{x^3}\right) < 0 \Rightarrow 1 < \frac{2}{x^3}$

$\Rightarrow x^3 < 2$ (if $x > 0$).

then: $0 < x < \sqrt[3]{2}$, and f is <u>downward</u> <u>concave</u>

<u>Table</u>.

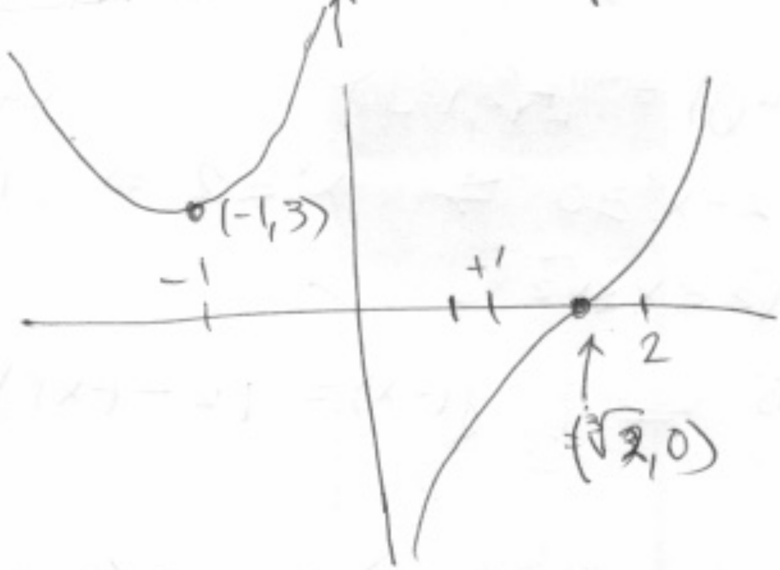
	$(-\infty, -1)$	$(-1, 0)$	$(0, \sqrt[3]{2})$	$(\sqrt[3]{2}, \infty)$
f'	< 0	> 0	> 0	> 0
f''	> 0	> 0	< 0	> 0
f	↘ and concave upwards	↗ concave upward.	↗ downward concave	↗ upwards concave

$x = -1$, $f(-1)$, local min.

$x_2 = \sqrt[3]{2}$, inflexion

6. local min: $(-1, f(-1)) = (-1, 3)$

Inflexion point: $(\sqrt[3]{2}, f(\sqrt[3]{2})) = (\sqrt[3]{2}, 0)$



$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\left(x^2 - \frac{2}{x}\right) = +\infty$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(x^2 - \frac{2}{x}\right) = -\infty$$

Asíntota vertical en $x = 0$

$$\lim_{x\to +\infty}\left(x^2 - \frac{2}{x}\right) = \infty$$

$$\lim_{x\to -\infty}\left(x^2 - \frac{2}{x}\right) = \infty$$

No hay asíntotas horizontales.

Quiz #7 - B | Cálculo Diferencial. | Viernes, 30 de junio de 2017

1. Graficar: $g(x) = (2-x^2)^{3/2}$

1. Dominio: $\mathrm{Dom}(g) = [-\sqrt{2}, \sqrt{2}]$.

We require $2 - x^2 \geq 0 \Rightarrow x^2 \leq 2 \Rightarrow |x| \leq \sqrt{2}$

$\Rightarrow -\sqrt{2} \leq x \leq \sqrt{2}$

Symmetries. The function is even: $g(-x) = (2-(-x)^2)^{3/2} = g(x)$.

2. Derivatives:

$$g'(x) = \frac{3}{2}(2-x^2)^{1/2}(-2x) = -3x(2-x^2)^{1/2}.$$

$$g''(x) = -3(2-x^2)^{1/2} - \frac{3x}{2}\frac{(-2x)}{(2-x^2)^{1/2}}$$

$$= -3\left(\frac{(2-x^2) - x^2}{(2-x^2)^{1/2}}\right) = -3\left(\frac{2-2x^2}{(2-x^2)^{1/2}}\right)$$

$$= -6\,\frac{(1-x^2)}{(2-x^2)^{3/2}}$$

3. Critical points (a) Boundaries: $x = -\sqrt{2}$, $x = +\sqrt{2}$

(b) $g'(x)$ always exists on $[-\sqrt{2}, \sqrt{2}]$

(c) $g'(x) = 0$ is $-3x(2-x^2)^{1/2} = 0$, i.e. $x = 0$, $x = \sqrt{2}$, $x = -\sqrt{2}$.

Then $\left.\begin{array}{l} x_1 = -\sqrt{2} \\ x_2 = 0 \\ x_3 = +\sqrt{2} \end{array}\right\}$ are the critical points

4. Increasing and decreasing intervals

$g'(x) = -3x(2-x^2)^{1/2} > 0$, if $x < 0$, $\Rightarrow g \searrow$

(since $(2-x^2)^{1/2} \geq 0$ always).

$g'(x) = -3x(2-x^2)^{1/2} < 0$, if $x > 0$. $\Rightarrow g \nearrow$.

5 Concavity. $g''(x) = -6\dfrac{(1-x^2)}{(2-x^2)^{3/2}}$

Since $\dfrac{-6}{(2-x^2)^{3/2}} < 0$ always, $g'' > 0$, if $1-x^2 < 0$.

i.e. $1 < x^2$, i.e: $\left\{\begin{array}{l} 1 < x \\ \text{or } x < -1 \end{array}\right.$ i.e. $\left\{\begin{array}{l} 1 < x < \sqrt{2} \\ -\sqrt{2} < x < -1 \end{array}\right\}$

g is upwards concave

Also, if $(1-x^2) > 0$, then $g'' < 0$,

i.e. $1 > x^2$, i.e., $|x| < 1$, i.e: $-1 < x < 1$

and g is downward concave

Table:

	$(-\sqrt{2}, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \sqrt{2})$
g'	<0	<0	>0	>0
g''	>0	<0	<0	>0
g	$\searrow$, upward	$\searrow$ downward	$\nearrow$ downward	$\nearrow$ upward

local mini $g(0)$.

Inflexion pts:
$x = -1$
$x = +1$

6. <u>local min</u> $(0, g(0)) = (0, 2^{3/2})$

<u>Inflexion points</u> $(1, g(1)) = (1, 1)$

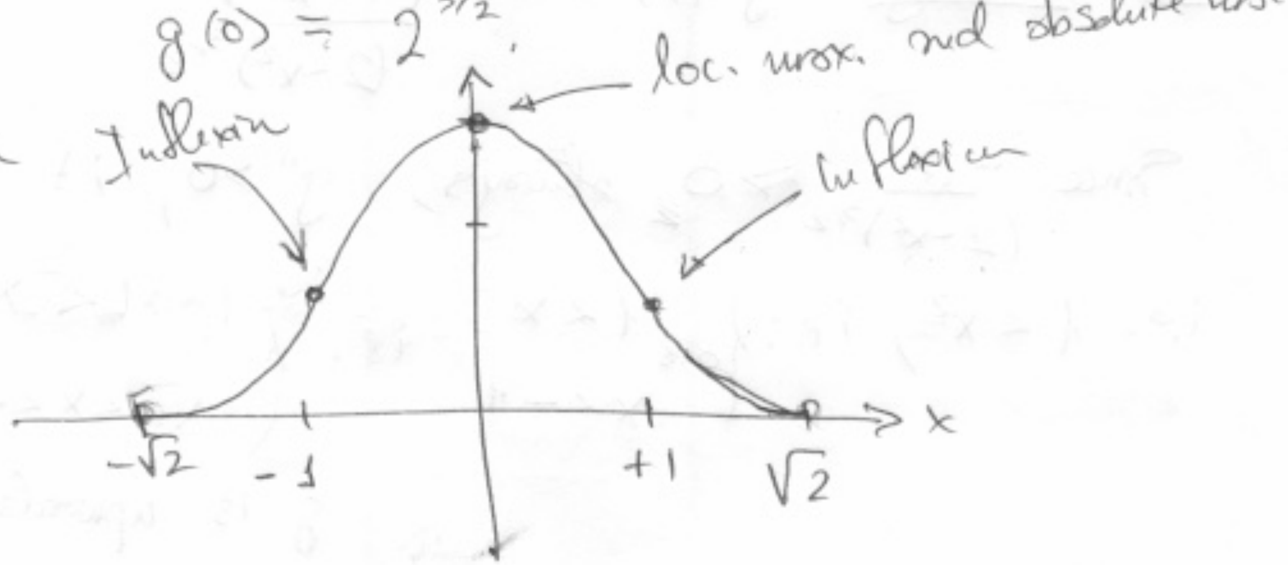
$(-1, g(-1)) = (-1, 1)$.

7. Since $\mathrm{Dom}(g) = [-\sqrt{2}, \sqrt{2}]$, there is <u>no</u> <u>asymptotes</u>;

$g(\sqrt{2}) = g(-\sqrt{2}) = 0$, $g(x) \geq 0$ always

$g(0) = 2^{3/2}$.

Graph



Quiz #7-C | Cálculo Diferencial | Viernes, 30/junio/2017

(1) Graficar $h(x) = x^{2/3}(x-5)$

(1) Domain and symmetries: $h(x)$ is not even neither odd

$Dom(h) = \mathbb{R}$

(2) Derivatives: since $h = x^{5/3} - 5x^{2/3}$

$h'(x) = \frac{5}{3}x^{2/3} - \frac{10}{3}x^{-1/3} = \frac{5}{3}\frac{(x-2)}{x^{1/3}}$, Pick at $x=0$

Also:

$h''(x) = \frac{10}{9}x^{-1/3} + \frac{10}{9}x^{-4/3} = \frac{10}{9}\frac{(x+1)}{x^{4/3}}$

(3) Critical points: (a) No boundaries

(b) $h'(x)$ does not exist at $x=0$.

(c) $h'(x) = 0$ at $x = 2$

Critical points $x=0$ and $x=2$.

(4) Increasing and decreasing. For $x \in (-\infty, 0)$,

$h'(x) = \frac{5}{3}\frac{(x-2)}{x^{1/3}} > 0$, since $\left.\begin{matrix} x-2<0 \\ x^{1/3}<0 \end{matrix}\right\} \Rightarrow h \nearrow$.

For $x \in (0,2)$

$h'(x) = \frac{5}{3}\frac{(x-2)}{x^{1/3}} < 0$, since $\left.\begin{matrix} x-2<0 \\ x^{1/3}>0 \end{matrix}\right\} \Rightarrow h \searrow$

For $x \in (2,\infty)$

$h'(x) = \frac{5}{3}\frac{(x-2)}{x^{1/3}} > 0$, since $\left.\begin{matrix} x-2>0 \\ x^{1/3}>0 \end{matrix}\right\} \Rightarrow h \nearrow$.

Then $f(0)$ is a local max

$f(2)$ is a local min

5. Concavity $h''(x) = \frac{10}{9}\frac{(x+1)}{x^{4/3}}$

$\frac{10}{9x^{4/3}} > 0$ always, except at $x=0$.

then, $h''(x) > 0$, if $x+1>0$ Upwards concave!
i.e. $x > -1$ (except at $x=0$)

$h''(x) < 0$, if $x+1<0$
ie $x<-1$ } Downwards concave.

Inflexion point at $x=-1$:

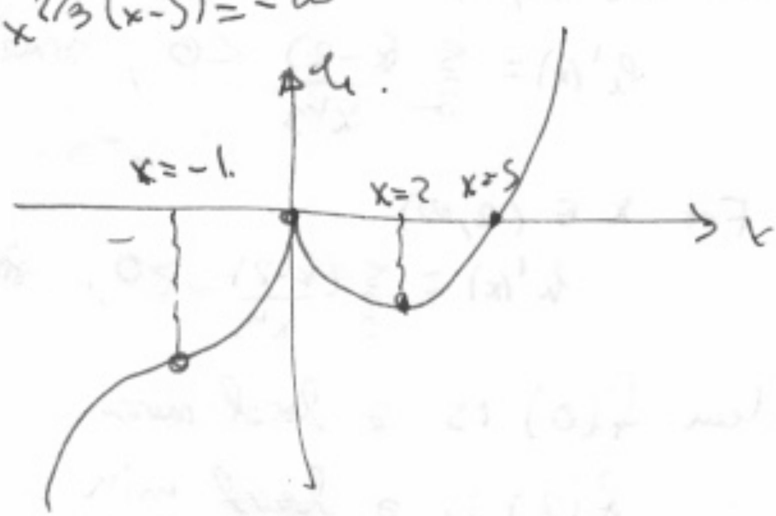
$h(0)=0$
$h(2) = 2^{2/3}(2-5) = -3(2^{2/3}) < 0$.

$h(x) > 0$, if $x > 5$
$h(x) < 0$, if $x < 5$.

(7) Since $Dom(h) = \mathbb{R}$ no vertical asymptotes.

$\lim_{x\to\infty} h(x) = \lim_{x\to\infty} x^{2/3}(x-5) = \infty$ No horizontal asymptote

$= \lim_{x\to-\infty} x^{2/3}(x-5) = -\infty$



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