Universidad Autónoma Metropolitana - Azcapotzalco | Viernes, Octubre 6, 2017

Quiz #1 Cálculo Integral

Nombre: ANSWER KEY.

1. Grafique el integrando y calcule la integral usando áreas:

$$\int_{-1}^{1} \left(1+\sqrt{1-x^2}\right) dx$$

2. Con los métodos vistos en clase, calcule:

$$\int_{1/2}^{1} 24\, u^2\, du$$

3. Usando sumas de Riemann, calcule

$$\int_{0}^{2} (2x+1)\, dx$$

SOLUCIONES

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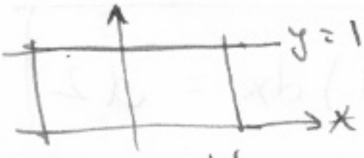
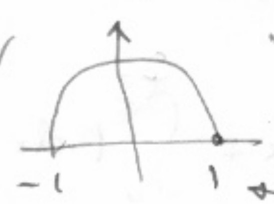
(1) Por propiedades de la integral:

$$\int_{-1}^{1} 1+\sqrt{1-x^2}\, dx = \int_{-1}^{1} 1\, dx + \int_{-1}^{1} \sqrt{1-x^2}\, dx = \boxed{2+\frac{\pi}{2}}$$, porque:

$$\int_{-1}^{1} 1\, dx = \text{Area}\left(\quad \right) = b \cdot h = 2 \cdot 1 = 2$$



$$\int_{-1}^{1} \sqrt{1-x^2}\, dx = \text{Area}\left(\quad \right) = \frac{\pi r^2}{2} = \frac{\pi}{2}.$$



= 1 =

③ $\int_{1/2}^{1} 24u^2\,du = 24\int_{1/2}^{1} u^2\,du = 24\left(\frac{1}{3}1^3 - \frac{1}{3}(1/2)^3\right)$

$= \frac{24}{3}\left(1 - \frac{1}{8}\right) = \frac{24}{3}\left(\frac{8-1}{8}\right) = \frac{24(8-1)}{24}$

$= \boxed{7}$

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③ $\int_0^2 (2x+1)\,dx = \lim_{N\to\infty} \sum_{k=1}^{N} (2x_k+1)\Delta x$, donde $\Delta x = \frac{2-0}{N} = \frac{2}{N}$

$x_k = x_0 + k\Delta x = \frac{2k}{N}$

$= \lim_{N\to\infty}\left(2\sum_{k=1}^{N} x_k \Delta x + \sum_{k=1}^{N} \Delta x\right)$

$= \lim_{N\to\infty}\left(2\sum_{k=1}^{N} \frac{2k}{N}\cdot\frac{2}{N} + \sum_{k=1}^{N}\frac{2}{N}\right)$

$= \lim_{N\to\infty}\left(\frac{8}{N^2}\sum_{k=1}^{N} k + \frac{2}{N}\sum_{k=1}^{N} 1\right)$

$= \lim_{N\to\infty}\left(\frac{8}{N^2}\,\frac{N(N+1)}{2} + \frac{2}{N}\cdot N\right)$

$= \lim_{N\to\infty}\left(4\left(1+\frac{1}{N}\right) + 2\right) = 4+2 = 6$

Entonces $\boxed{\int_0^2 (2x+1)\,dx = 6}$

Verificando:

$\int_0^2 (2x+1)\,dx = 2\int_0^2 x\,dx + \int_0^2 1\cdot dx = 2\frac{2^2}{2} + 2\cdot 1 = 4+2$

$= 6$ ✓

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