

Quiz #2. CÁLCULO INTEGRAL

2017.

Nombre: **ANSWER KEY**

1. Evalúa, $\int_0^{\pi/2} \theta^2 d\theta$

2. Calcule la integral: $\int_{-\pi/3}^{-\pi/4} \left(4 \sec^2(t) + \frac{\pi}{t^2} \right) dt$

3. Calcule la derivada de: $\Phi(x) = \int_2^{2x^2} (t-2) \tan^2(1+t^2) dt$

SOLUCIONES

① De acuerdo a lo estudiado con sumas de Riemann.

$$\int_0^{\pi/2} \theta^2 d\theta = \int_a^b x^2 dx = \frac{b^3}{3} - \frac{a^3}{3} = \frac{1}{3} \left(\frac{\pi}{2} \right)^3 - \frac{0^3}{3} = \frac{\pi^3}{24} \quad \checkmark$$

② Por linealidad de la integral:

$$\begin{aligned} \int_{-\pi/3}^{-\pi/4} 4 \sec^2(t) + \frac{\pi}{t^2} dt &= 4 \int_{-\pi/3}^{-\pi/4} \sec^2(t) dt + \pi \int_{-\pi/3}^{-\pi/4} \frac{1}{t^2} dt \\ &= 4 \tan(t) \Big|_{-\pi/3}^{-\pi/4} + \pi \left(-\frac{1}{t} \right) \Big|_{-\pi/3}^{-\pi/4} \quad \text{por el Teorema Fundamental del Cálculo.} \\ &= 4 \left(\tan\left(-\frac{\pi}{4}\right) - \tan\left(-\frac{\pi}{3}\right) \right) + \pi \left(\frac{4}{\pi} - \frac{3}{\pi} \right) \end{aligned}$$

= 2 =

$$= 4((-1) - (-\sqrt{3})) + (4-3) = 4(\sqrt{3} - 1) + 1$$



Asi:

$$\alpha^2 + \left(\frac{1}{2}\right)^2 = 1 \Rightarrow \alpha^2 = \frac{3}{4} \Rightarrow \alpha = \frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\int_{-\pi/3}^{-\pi/4} 4 \sec^2 t + \frac{\pi}{t^2} dt = 4\sqrt{3} - 3$$

3) Calcular

$$\frac{d\Phi}{dx} = \frac{d}{dx} \left(\int_2^{2x^2} (t-2) \tan^2(1+t^2) dt \right) =$$

$$y = y(x) = 2x^2$$

$$= \frac{d}{dy} \left(\int_2^y (t-2) \tan^2(1+t^2) dt \right) \cdot \frac{dy}{dx}$$

Por la regla de la cadena.

$$= (y-2) \tan^2(1+y^2) \cdot \frac{d}{dx} (2x^2)$$

$$= (2x^2-2) \tan^2(1+(2x^2)^2) \cdot (4x)$$

$$\frac{d\Phi}{dx} = 8x(x^2-1) \tan^2(1+4x^4)$$