UNIVERSIDAD AUTÓNOMA METROPOLITANA - AZCAPOTZALCO

CÁLCULO INTEGRAL | ~~[illegible]~~ #1 | Jueves, Octubre 26, 2017

Nombre: QUIZ #4

① Encuentre la antiderivada de $f(x) = x^3 \ln x$

② Encuentre la integral $\int_0^1 x \sqrt{1-x}\, dx$.

(Hint: Use alguna sustitución)

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① La antiderivada de $f(x)$ es la integral indefinida:

$$\int x^3 \ln x\, dx = \int \left(\tfrac{1}{4}x^4\right)' \ln x\, dx$$

$$= \tfrac{1}{4}x^4 \ln x - \int \tfrac{1}{4}x^4 (\ln x)'\, dx, \quad \text{por integración por partes}$$

$$= \tfrac{1}{4}x^4 \ln x - \tfrac{1}{4}\int x^4 \cdot \tfrac{1}{x}\, dx$$

$$= \tfrac{1}{4}x^4 \ln x - \tfrac{1}{4}\int x^3\, dx$$

$$= \tfrac{1}{4}x^4 \ln x - \tfrac{1}{4}\left(\tfrac{1}{4}x^4\right) + C$$

I.e.

$$\boxed{\int x^3 \ln x\, dx = \tfrac{1}{4}x^4\left(\ln x - \tfrac{1}{4}\right) + C}$$

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② Tomar $u = \sqrt{1-x}$. Entonces: $\frac{du}{dx} = -\frac{1}{2\sqrt{1-x}} = \frac{-1}{2u}$

Entonces: $dx = -2u\, du$, y $x = 1-u^2$.

$= \int =$

Así: $\displaystyle\int_0^1 x\sqrt{1-x^2}\,dx = \int_1^0 (1-u^2)\,u\,(-2u\,du)\,du$

pues $u(1)=\sqrt{1-1}=0$
$u(0)=\sqrt{1-0}=1.$

$$= -2\int_1^0 (1-u^2)u^2\,du = 2\int_0^1 (u^2-u^4)\,du = 2\left(\frac{u^3}{3}-\frac{u^5}{5}\right)\Big|_0^1$$

$$= 2\left(\frac{1}{3}-\frac{1}{5}\right) = 2\,\frac{(5-3)}{15} = \frac{4}{15}$$

$$\boxed{\int_0^1 x\sqrt{1-x^2}\,dx = \frac{4}{15}}$$

Nota: También funciona el cambio de variable

$$u = 1-x$$

Entonces $x = 1-u$, y $\frac{du}{dx} = -1$:

Así $u(1)=0$ y además.
$u(0)=1$

$$\int_0^1 x\sqrt{1-x}\,dx = \int_1^0 (1-u)\,u^{1/2}(-1)\,du = -\int_1^0 (1-u)u^{1/2}\,du$$

$$= \int_0^1 (1-u)u^{1/2}\,du = \int_0^1 u^{1/2}-u^{3/2}\,du = \left(\frac{2}{3}u^{3/2}-\frac{2}{5}u^{5/2}\right)\Big|_0^1$$

$$= \left(\frac{2}{3}-\frac{2}{5}\right) = 2\left(\frac{1}{3}-\frac{1}{5}\right) = 2\,\frac{(5-3)}{15} = \frac{4}{15}$$

Mismo resultado.