UNIVERSIDAD AUTÓNOMA METROPOLITANA - AZCAPOTZALCO

Examen #1 CÁLCULO INTEGRAL | Viernes 27 de octubre de 2017

Soluciones

1 (a) Por el Teorema Fundamental del Cálculo, la derivada de

$$\Phi(y) = \int_0^y \tan^4(s)\,ds \quad \text{es} \quad \frac{d\Phi}{dy} = \tan^4(y)$$

(b) Por la regla de la cadena.

$$\frac{d\Psi}{dx} = \left.\frac{d\Phi}{dy}\right|_{y=y(x)} \cdot \frac{dy}{dx} = \left.\tan^4(y)\right|_{y=y(x)} \frac{d}{dx} x^5 = \boxed{5x^4 \tan^4(x^5)}$$

2 Aquí, tenemos que calcular: $(n=4)$.

$$\sum_{k=1}^{n=4} f(x_k)\Delta x_k = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$$

$$\Delta x = \frac{b-a}{n} = \frac{5-1}{4} = 1.$$

$$x_k = 1 + k\Delta x = 1 + k \Rightarrow \begin{cases} x_1 = 2 \\ x_2 = 3 \\ x_3 = 4 \\ x_4 = 5. \end{cases}$$

Entonces, como $f(x) = \frac{1}{x}$

$$\sum_{k=1}^{4} f(x_k)\Delta x = \frac{1}{x_1}\cdot 1 + \frac{1}{x_2}\cdot 1 + \frac{1}{x_3}\cdot 1 + \frac{1}{x_4}\cdot 1$$

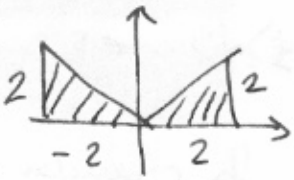
$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{60+40+30+24}{120}$$

$$= \boxed{\frac{154}{120}}$$

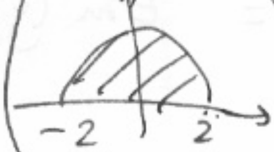
$= 1 =$

③ Como la integral es una operación lineal:

$$\int_{-2}^{2}\left(-3|x| + \sqrt{4-x^2}\right)dx = -3\int_{-2}^{2}|x|\,dx + \int_{-2}^{2}\sqrt{4-x^2}\,dx$$

Como $\displaystyle\int_{-2}^{2}|x|\,dx = \text{Área}\Big($  $\Big) = \frac{2\cdot 2}{2} + \frac{2\cdot 2}{2} = 4$

$\frac{b\cdot h}{2} + \frac{b\cdot h}{2}$

y $\displaystyle\int_{-2}^{2}\sqrt{4-x^2}\,dx = \text{Área}\Big($  $\Big) = \frac{\pi\cdot 2^2}{2} = 2\pi$

Entonces:

$$\int_{-2}^{2}\left(-3|x| + \sqrt{4-x^2}\right)dx = -3(4) + 2\pi = \boxed{-12 + 2\pi}$$

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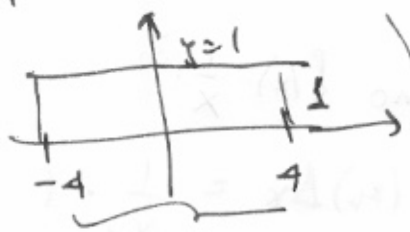
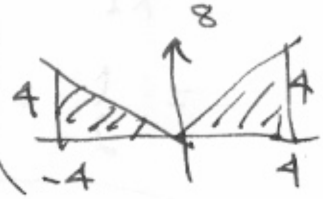
④ Como la integral es una operación lineal:

$$\int_{-4}^{4}\left(3 - \tfrac{1}{2}|x|\right)dx = 3\int_{-4}^{4}1\,dx - \frac{1}{2}\int_{-4}^{4}|x|\,dx$$

Como $\displaystyle\int_{-4}^{4}1\cdot dx = \text{Área}\Big($  $\Big) = 8\cdot 1 = 8$

y $\displaystyle\int_{-4}^{4}|x|\,dx = \text{Área}\Big($  $\Big) = \frac{4\cdot 4}{2} + \frac{4\cdot 4}{2} = 16$

Entonces

$$\int_{-4}^{4}\left(3 - \tfrac{1}{2}|x|\right)dx = 3\cdot 8 - \frac{1}{2}\cdot 16 = 24 - 8 = \boxed{16}$$

=2=

(5) Como $\int_1^9 f(x)\,dx = 6$; $\int_7^9 f(x)\,dx = 6$; $\int_7^9 h(x)\,dx = -2$.

Entonces:

$$\int_1^7 f(x)\,dx = \int_1^9 f(x)\,dx + \int_9^7 f(x)\,dx,$$

$$= \int_1^9 f(x)\,dx - \int_7^9 f(x)\,dx$$

$$= 6 - 6 = 0.$$

Entonces:

$$\int_7^9 (2f(x) - 3h(x))\,dx + \int_1^7 f(x)\,dx =$$

$$= 2\int_7^9 f(x)\,dx - 3\int_7^9 h(x) + \int_1^7 f(x)\,dx = 2\cdot 6 - 3(-2) + 0$$

$$= 12 + 6 + 0 = \boxed{18}.$$

(6) Evalúe:

$$\int_0^{\pi/4} \left(8\sec^2 t + \frac{\operatorname{Log}(2)}{t^2}\right) dt = 8\int_0^{\pi/4} \sec^2 t\,dt + \operatorname{Log}(2)\int_0^{\pi/4} \frac{1}{t^2}\,dt.$$

$$= 8 \tan t \Big|_0^{\pi/4} + \operatorname{Log}(2)\left(-\frac{1}{t}\right)\Big|_0^{\pi/4}$$ Hasta aquí, está bien:

$$= 8\left(\tan\left(\frac{\pi}{4}\right) - \tan 0\right)$$

$$= 8(1-0) + \operatorname{Log} 2\left(-\frac{4}{\pi}\right) + \frac{\operatorname{Log}(2)}{t}\Big|_{t=0}$$

$= 3 =$

$$\int_0^{\pi/4}\left(8\sec^2 t\, dt + \frac{\log(2)}{t^2}\right)dt = 8 - \frac{4}{\pi}\log(2) + \frac{\log(2)}{t}\Big|_{t=0}$$

No está definida

Luego veremos integrales impropias

$\int_0^7 \frac{1}{t^2}\, dt$ es una integral impropia

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(7) Encuentre la antiderivada de. $\sqrt{x}\sin^2(x^{3/2}-1)$.

$$\int \sqrt{x}\sin^2(x^{3/2}-1)\,dx =$$

$u = x^{3/2} - 1$

$\frac{du}{dx} = \frac{3}{2}\sqrt{x}$

Entonces $\frac{2}{3}du = \sqrt{x}\,dx$

$$= \int \sin^2(u)\frac{2}{3}\,du = \frac{2}{3}\int \frac{1-\cos 2u}{2}\,du$$

$$= \frac{1}{3}\int (1-\cos 2u)\,du = \frac{1}{3}\left(u - \frac{\sin(2u)}{2}\right) + C.$$

$$= \boxed{\frac{1}{3}\left((x^{3/2}-1) - \frac{1}{2}\sin(2(x^{3/2}-1))\right) + C}$$

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(8) Encuentre la antiderivada de $\left(1-\cos\left(\frac{t}{2}\right)\right)^2\sin\left(\frac{t}{2}\right)$.

$$\int \left(1-\cos\left(\frac{t}{2}\right)\right)^2 \sin\left(\frac{t}{2}\right)dt.$$

Take: $u = 1 - \cos\left(\frac{t}{2}\right)$

Then $\frac{du}{dt} = \frac{1}{2}\sin\left(\frac{t}{2}\right)$

Then $2\,du = \sin\left(\frac{t}{2}\right)dt$.

$$\int \left(1-\cos\left(\tfrac{t}{2}\right)\right)^2 \sin\left(\tfrac{t}{2}\right) dt = \int u^2 \, 2\, du = 2\int u^2 du$$

$$= 2\,\frac{u^3}{3} + C = \boxed{\frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3 + C}$$

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(9) $\int \sin(\ln x)\, dx$.

Usamos el cambio de variable $u = \ln x$.

Entonces $x = e^u$ y $\frac{du}{dx} = \frac{1}{x}$

Así $\int \sin(\ln x)\, dx = \int \sin(u)\, e^u\, du$ $\left(\Rightarrow dx = x\, du = e^u du\right)$

Integrando por partes: dos veces:

$$= e^u \sin u - \int e^u \cos u\, du = e^u \sin u - \left[e^u \cos u - \int e^u(-\sin u) du\right]$$

$$= e^u \sin u - e^u \cos u - \int e^u \sin u\, du$$, obtenemos la misma integral. Entonces, despejando esta integral:

$$2\int \sin(u)\, e^u\, du = e^u(\sin u - \cos u) + C$$

i.e. $$\int \sin(\ln x)\, dx = \frac{1}{2} e^u(\sin u - \cos u) + C$$

y regresando a la variable original:

$$\boxed{\int \sin(\ln x)\, dx = \frac{1}{2} x\left(\sin(\ln x) - \cos(\ln x)\right) + C}$$

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⑩ Evalúe: $\int x^3 e^{2x}\,dx$

Necesitamos integrar por partes tres veces para que, al derivar el monomio x^3, ir bajando su potencia:

$$\int x^3 e^{2x}\,dx = x^3 \frac{1}{2} e^{2x} - \int 3x^2\left(\frac{1}{2}e^{2x}\right)dx$$, se integró por partes.

$$= \frac{1}{2}x^3 e^{2x} - \frac{3}{2}\left[\int x^2 e^{2x}\,dx\right]$$

$$= \frac{1}{2}x^3 e^{2x} - \frac{3}{2}\left[x^2\left(\frac{1}{2}e^{2x}\right) - \int (2x)\left(\frac{1}{2}e^{2x}\right)dx\right]$$

se integró por partes nuevamente:

$$= \frac{1}{2}x^3 e^{2x} - \frac{3}{2^2}x^2 e^{2x} + \frac{3}{2}\left[\int x e^{2x}\,dx\right]$$

e integrando por partes nuevamente:

$$= \frac{1}{2}x^3 e^{2x} - \frac{3}{2^2}x^2 e^{2x} + \frac{3}{2}\left[x\left(\frac{1}{2}e^{2x}\right) - \int 1\cdot\frac{1}{2}e^{2x}\,dx\right]$$

$$= \frac{1}{2}x^3 e^{2x} - \frac{3}{4}x^2 e^{2x} + \frac{3}{4}x e^{2x} - \frac{3}{4}\int e^{2x}\,dx$$

Así:

$$\int x^3 e^{2x}\,dx = \frac{1}{2}x^3 e^{2x} - \frac{3}{4}x^2 e^{2x} + \frac{3}{4}x e^{2x} - \frac{3}{8}e^{2x} + C$$

i.e.

$$\boxed{\int x^3 e^{2x}\,dx = \left(\frac{4x^3 - 6x^2 + 6x - 3}{8}\right)e^{2x} + C}$$