UNIVERSIDAD AUTÓNOMA METROPOLITANA – AZCAPOTZALCO. 2017

CÁLCULO INTEGRAL. Quiz – Tarea #6 | Viernes 10 de noviembre de

Nombre:

---

(1) Calcule la integral $\int \sin^5 x \cos^3 x \, dx$. Hint: Separe un $\sin x$ y use $\sin^2 x = 1 - \cos^2 x$

---

(2) Calcule la integral $\int \frac{\sqrt{y^2 - 49}}{y} \, dy$. Hint: $y = a \sin\theta$, $y = a \tan\theta$, $y = a \sec\theta$

SOLUCIONES

(1) $\int \sin^5 x \cos^3 x \, dx = \int \sin^4 x \cos^3 x (\sin x) \, dx$

$= \int (\sin^2 x)^2 \cos^3 x (-\cos x)' \, dx = \int (1 - \cos^2 x)^2 \cos^3 x (-\cos x)' dx$

$u = -\cos x$. $\frac{du}{dx} = \sin x$.

$= \int (1 - u^2)^2 (-1)^3 u^3 \, du = -\int (1 - 2u^2 + u^4) u^3 \, du$

$= -\int u^7 - 2u^5 + u^3 \, du = -\left(\frac{1}{8} u^8 - \frac{2}{6} u^6 + \frac{1}{4} u^4 \right) + C$

$= -\frac{1}{8} \cos^8 x + \frac{1}{3} \cos^6 x - \frac{1}{4} \cos^4 x + C$

---

(2) Debemos usar $y = 7 \sec\theta$

$\frac{dy}{d\theta} = 7 \sec\theta \tan\theta$

Así:

$\int \frac{\sqrt{y^2 - 49}}{y} \, dy = \int \frac{\sqrt{49 \sec^2\theta - 49}}{7 \sec\theta} \cdot 7 \sec\theta \tan\theta \, d\theta$

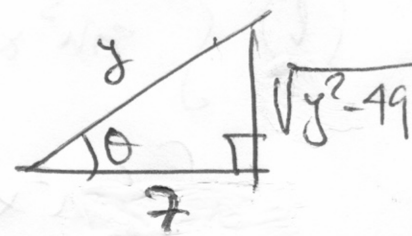
$$= \int \sqrt{49(\sec^2\theta - 1)} \cdot \tan\theta \, d\theta$$

$$= 7 \int \sqrt{\tan^2\theta} \, \tan\theta \, d\theta = 7 \int |\tan\theta| \tan\theta \, d\theta$$

Si $\theta \in [0, \frac{\pi}{2})$, $\tan\theta \geq 0$, así $|\tan\theta| = \tan\theta$

$$= 7 \int \tan^2\theta \, d\theta = 7 \int (1+\tan^2\theta) - 1 \, d\theta = \int (\sec^2\theta - 1) d\theta$$

$$= 7(\tan\theta - \theta) + C =$$



Pero:

$$\frac{1}{\cos\theta} = \sec\theta = \frac{y}{7} \Rightarrow \cos\theta = \frac{7}{y} \Rightarrow$$

Así: $\sin\theta = \dfrac{\sqrt{y^2-49}}{y}$ y $\tan\theta = \dfrac{\sqrt{y^2-49}}{7}$.

Además $\theta = \operatorname{Arccos}\left(\frac{7}{y}\right)$.

Por tanto:

$$\int \frac{\sqrt{y^2-49}}{y} \, dy = 7\left(\frac{\sqrt{y^2-49}}{7} - \operatorname{Arccos}\left(\frac{7}{y}\right) + C\right)$$

i.e.

$$\boxed{\int \frac{\sqrt{y^2-49}}{y} \, dy = \sqrt{y^2-49} - 7 \operatorname{Arccos}\left(\frac{7}{y}\right) + C}$$