...-Azcapotzalco Cálculo Integral | Viernes, 17 noviembre, 2017

ANSWER KEY Examen #2

(1) We must compute: $\int \sin^2(2\theta)\cos^3(2\theta)\,d\theta =$

$$= \int \sin^2(2\theta)\cos^2(2\theta)\cos(2\theta)\,d\theta = \int \sin^2(2\theta)(1-\sin^2 2\theta)\frac{(\sin 2\theta)'}{2}\,d\theta$$

$u = \sin(2\theta)$

$$= \int u^2(1-u^2)\frac{1}{2}\,du = \frac{1}{2}\int u^2 - u^4\,du = \frac{1}{2}\left(\frac{u^3}{3} - \frac{u^5}{5}\right) + C$$

Then, $\boxed{\int \sin^2 2\theta \cos^3(2\theta)\,d\theta = \frac{\sin^3(2\theta)}{6} - \frac{\sin^5(2\theta)}{10} + C}$

(2) We need to calculate: $\int 3\sec^4 3x\,dx = 3\int \sec^2(3x)\sec^2(3x)\,dx$

$u = \tan(3x)$

$$= 3\int (1+\tan^2(3x))\frac{(\tan(3x))'}{3}\,dx = 3\int (1+u^2)\frac{1}{3}\,du$$

$$= \int (1+u^2)\,du = u + \frac{u^3}{3} + C = \tan(3x) + \frac{1}{3}\tan^3(3x) + C$$

Then, we have $\boxed{\int 3\sec^4(3x)\,dx = \tan(3x) + \frac{1}{3}\tan^3(3x) + C}$

(3) $\int \frac{5}{\sqrt{25x^2-9}}\,dx = \frac{5}{3}\int \frac{1}{\sqrt{\left(\frac{5}{3}x\right)^2 - 1}}\,dx, \quad x > \frac{3}{5}$

Let $\frac{5}{3}x = \sec\theta$; Then, $\frac{5}{3}\frac{dx}{d\theta} = \sec\theta\tan\theta$

$= 1 =$

nce:

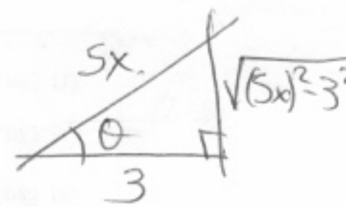
$$= \frac{5}{3}\int \frac{1}{\sqrt{\sec^2\theta - 1}}\,\frac{3}{5}\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{1}{|\tan\theta|}\sec\theta\tan\theta\,d\theta.$$

If $\theta \in (0, \frac{\pi}{2})$, $\tan\theta > 0$, then $|\tan\theta| = \tan\theta$.

Hence:

$$= \int \sec\theta\,d\theta = \log|\sec\theta + \tan\theta| + C$$



Now $\frac{1}{\sec\theta} = \frac{3}{5x}$ i.e. $\cos\theta = \frac{3}{5x}$

Hence $\tan\theta = \frac{\sqrt{(5x)^2 - 3^2}}{3}$. Therefore:

$$\boxed{\int \frac{5}{\sqrt{25x^2-9}}\,dx = \log\left|\frac{\sqrt{25x^2-9}}{3} + \frac{5}{3}x\right| + C}$$

Remember $\frac{d}{d\theta}\left(\log|\sec\theta + \tan\theta|\right) = \frac{(\sec\theta + \tan\theta)'}{\sec\theta + \tan\theta} =$

$$= \frac{\sec\theta\tan\theta + \sec^2\theta}{\sec\theta + \tan\theta} = \frac{(\tan\theta + \sec\theta)\sec\theta}{\tan\theta + \sec\theta} = \sec\theta$$

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(4) We have to evaluate:

$$\int_1^3 \frac{1}{(x^2 - 4x + 4)^{1/3}}\,dx = \int_1^3 \frac{dx}{((x-2)^2)^{1/3}} =$$

$$= \int_1^3 \frac{dx}{(x-2)^{2/3}},$$ which is an improper integral

changing the variable $y = x-2 = \varphi(x)$

Then $y = \varphi(1) = 1-2 = -1$

$y = \varphi(3) = 3-2 = 1$,

we have:

$$= \int_{-1}^{1} \frac{dy}{y^{2/3}} = 2\int_{0}^{1} \frac{dy}{y^{2/3}}$$, since it is an even function.

$$= 2 \lim_{\epsilon \to 0^+} \int_{\epsilon}^{1} \frac{dy}{y^{2/3}}$$, since it is an improper integral.

$$= 2 \lim_{\epsilon \to 0^+} \int_{\epsilon}^{1} y^{-2/3}\, dy = 2 \lim_{\epsilon \to 0^+} 3y^{1/3} \Big|_{\epsilon}^{1} =$$

$$= 6 \lim_{\epsilon \to 0^+} 1 - \epsilon^{1/3} = 6 \cdot 1$$; thence:

$$\boxed{\int_{1}^{3} \frac{1}{(x^2-4x+4)^{1/3}}\, dx = 6}$$

5 We need to find: $\int \frac{1}{(1+x)(1+x^2)} dx$.

We require partial fractions:

$$\frac{1}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{1+x^2} = \frac{A(1+x^2) + (1+x)(Bx+C)}{(1+x)(1+x^2)}$$

Hence: $1 = A(1+x^2) + (1+x)(Bx+C)$.

Take: $x = -1$ $\quad 1 = A(1+1) + 0 \Rightarrow \boxed{A = \frac{1}{2}}$

take $\underline{x=0}$; $1 = A + 1 \cdot C \Rightarrow C = 1 - A$

$C = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow \boxed{C = \frac{1}{2}}$

Take $\underline{x = 1}$

$1 = 2A + 2(B+C)$

$1 = 1 + 2(B+C)$ $\boxed{B = -\frac{1}{2}}$

$B + C = 0 \Rightarrow B = -C = -\frac{1}{2}$

thus:

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \int \frac{1/2}{1+x} + \frac{-1/2\,x + 1/2}{1+x^2}\,dx.$$

$$= \frac{1}{2}\int \frac{1}{1+x}\,dx - \frac{1}{2\cdot 2}\int \frac{2x}{1+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$

$$= \frac{1}{2}\log|1+x| - \frac{1}{4}\log(1+x^2) + \frac{1}{2}\operatorname{Arctan} x + C$$

i.e.

$$\boxed{\int \frac{1}{(1+x)(1+x^2)}\,dx = \log\left(\frac{(|1+x|)^{1/2}}{(1+x^2)^{1/4}}\right) + \frac{1}{2}\operatorname{Arctan} x + C}$$