UAM - AZCAPOTZALCO

Viernes, Feb 2, 2018

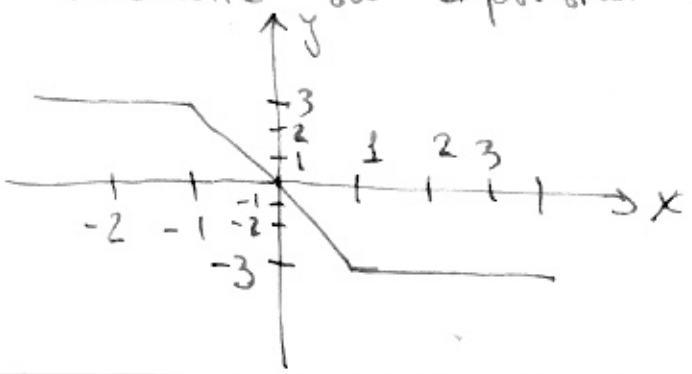
INTRO AL CÁLCULO Quiz #2

Nombre:

(1) Encuentre el dominio de la función y su rango.

$$f(x) = -\sqrt{25 - x^2}$$

(2) Si la gráfica de una función es como sigue, encuentre su expresión algebraica:



SOLUTION SET

(1) To get the domain, we require: $25 - x^2 \geq 0$ for the root to make sense. Then:

$$0 \leq 25 - x^2$$

$$0 \leq x^2 \leq 25$$

$$0 \leq |x| \leq 5 \Rightarrow -5 \leq x \leq 5$$

Then $\boxed{\text{Dom}(f) = [-5, 5]}$

To get the range, notice that $f(x) = -\sqrt{25 - x^2} \leq 0$

$$\Rightarrow$$

But, $f(x)$ can be at least $f(0) = -\sqrt{25-0} = -5$

then $-5 \le f(x) \le 0$ and so,

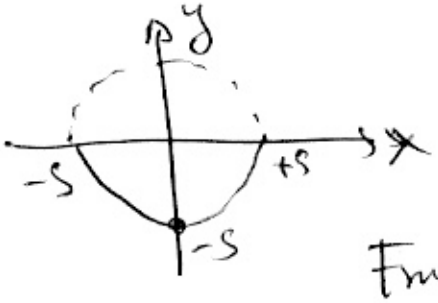
$$\boxed{\text{Ran}(f) = [-5, 0]}$$

Also, the graph of $f(x)$ represents a semi circle

$$y = -\sqrt{25 - x^2}$$

$$y^2 = 25 - x^2$$

$x^2 + y^2 = 25$, circle of radius 5 and center at $(0,0)$



The lower part of the circle is considered since $f(x) \le 0$.

From the picture: $\text{Dom}(f) = [-5, 5]$

$\text{Ran}(f) = [-5, 0]$

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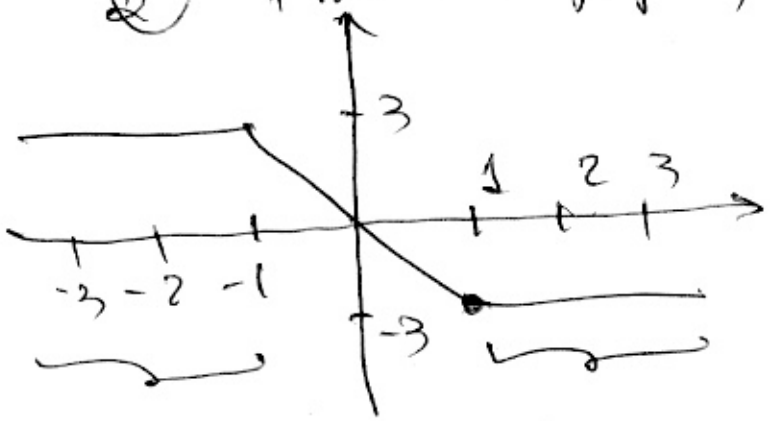
(2) From the graph, we observe



$f(x) = +3$ $\quad$ $f(x) = -3$

$f(x) = -3x.$

Then, the expression of $f(x)$ is a piece-wise defined function:

$$f(x) = \begin{cases} 3, & x \le -1 \\ -3x, & -1 \le x \le 1 \\ -3, & x > +1 \end{cases}$$