UAM - Azcapotzalco. | Viernes, Febrero 2, 2018

ECUACIONES DIFERENCIALES ORDINARIAS   Quiz #2

Nombre:

(1) Resuelva el problema de valores iniciales:

$$\frac{dy}{dt} = \frac{1-y^2}{y}, \quad y(0) = -2$$

(2) Encuentre la función $g(y)$ en la siguiente Ec.Dif, si sabemos que $y(t) = e^{2t}$ es su solución:

$$\frac{dy}{dt} = 2y - t + g(y)$$

SOLUTION SET

(1) This is a separable equation. Then:

$$\int \frac{y}{1-y^2}\,dy = \int dt \; ;$$

Or $\boxed{y(t) = \pm\sqrt{1+C_4 e^{-2t}}}$

i.e.

$$\int \frac{d}{dy}\left(-\frac{1}{2}\left(\log|1-y^2|\right)\right)dy = t + C_1$$

$$-\frac{1}{2}\operatorname{Log}|1-y^2| = t + C_1$$

$$\operatorname{Log}|1-y^2| = -2t + C_2$$

$$1 - y^2 = C_3 e^{-2t}$$

$$\boxed{y^2(t) = 1 + C_4 e^{-2t}}$$ Implicit solution

Using the initial condition: $y(0) = -2 < 0$, we used the negative sign:

$$y(t) = -\sqrt{1+C_4 e^{-2t}}$$

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Evaluating at $t=0$ to get C_4:

$$-2 = y(0) = -\sqrt{1+C_4 e^{0}} = -\sqrt{1+C_4}$$

$$\Rightarrow 2 = \sqrt{1+C_4} \Rightarrow 4 = 1+C_4 \Rightarrow C_4 = 3$$

$$\Rightarrow \boxed{y(t) = -\sqrt{1+3e^{-2t}}}$$

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(2) The solution $y(t) = e^{2t}$, solves:

$$\frac{dy}{dt} = 2y - t + g(y).$$

Then, ~~solve~~ substitute $y(t) = e^{2t}$ into the equation, and should hold since it is solution:

$$\frac{d}{dt}(e^{2t}) = 2e^{2t} - t + g(y)$$

$$2e^{2t} = 2e^{2t} - t + g(y)$$

$$0 = -t + g(y)$$

Then:

$$g(y) = +t = +\frac{1}{2}\log y \Rightarrow \boxed{g(y) = \frac{1}{2}\log y}$$

since $\left[\begin{array}{l} y = e^{+2t} \\ \log y = +2t \\ +\frac{1}{2}\log y = t \end{array}\right]$

The Diff. Eq is thus

$$\boxed{\frac{dy}{dt} = 2y - t + \frac{1}{2}\log y}$$