UAM - AZCAPOTZALCO | ECUACIONES DIFERENCIALES | Miércoles 14 de febrero 2018

Quiz #4 | Nombre: SOLUTION SET.

JUSTIFIQUE y DESARROLLE sus RESPUESTAS. EXPLIQUE mientras hace su desarrollo para obtener puntaje

(1) Bosqueje la línea fase de la Ec. Diferencial.

$$\frac{dy}{dt} = \frac{1}{(y-4)(y-1)}$$

y bosqueje la gráfica de la solución con condición inicial $y(0) = \frac{5}{2}$ (Incluya valores negativos de t).

(2) Resuelva la Ecuación Diferencial:

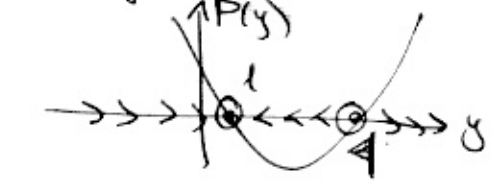
$$t\frac{dy}{dt} + 3(y+t^2) = \frac{\sin t}{t}$$

SOLUTION SET

(1). The Diff Eq. does not make any sense at $y=1$, $y=4$.

We have in the denominator the polynomial.

$P(y) = (y-4)(y-1) = y^2 - 5y + 4$,

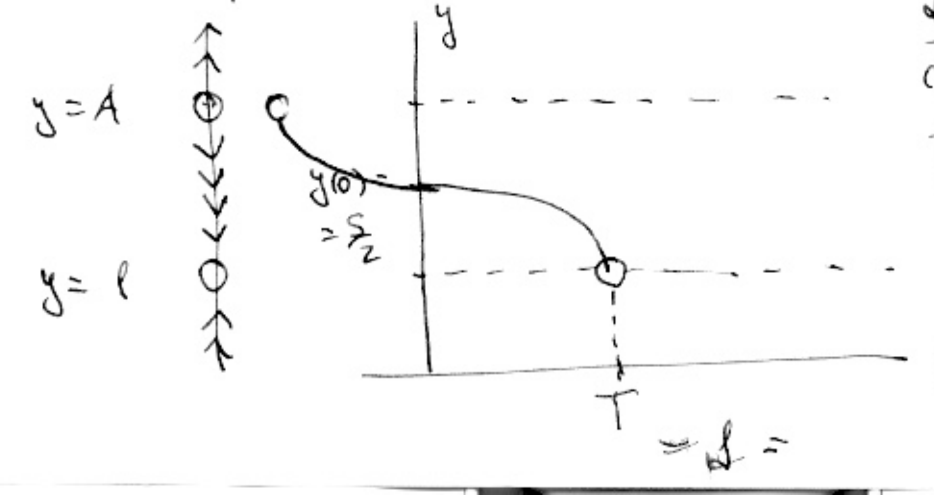


$P(y) > 0$ in $(-\infty, 1) \cup (4, \infty)$

$P(y) < 0$ in $(1, 4)$ Then: $\frac{dy}{dt} > 0$ in $(-\infty, 1) \cup (4, \infty)$

Then the phase line is:

$\frac{dy}{dt} < 0$ in $(1, 4)$.



The solution never hits $y=1$, but

$\lim_{t \to T} y(t) = 1$

for some $T < \infty$

(2) Solve $t\frac{dy}{dt} + 3(y+t^2) = \frac{\sin t}{t}$.

Write the Diff. Eq in standard form: $t\frac{dy}{dt} + 3y = \frac{\sin t}{t} - 3t^2$.

$$\Rightarrow \boxed{\frac{dy}{dt} + \frac{3}{t}y = \frac{\sin t}{t^2} - 3t}$$

The integrating factor is: $\mu(t) = e^{\int \frac{3}{t}dt} = e^{3\log|t|} = t^3$.

Now, we must solve:

$$\int \mu(t)b(t)dt = \int t^3\left(\frac{\sin t}{t^2} - 3t\right)dt = \int t\sin t - 3t^4 dt$$

$$= -t\cos t + \int \cos t - \frac{3}{5}t^5 = -t\cos t + \sin t - \frac{3}{5}t^5.$$

Then, the solution is: $y(t) = \frac{1}{\mu(t)}\int \mu(t)b(t)dt + \frac{C}{\mu(t)}$

$$y(t) = \frac{1}{t^3}\left(-t\cos t + \sin t - \frac{3}{5}t^5\right) + \frac{C}{t^3}$$

ie.

$$\boxed{y(t) = \frac{\sin t}{t^3} - \frac{\cos t}{t^2} - \frac{3}{5}t^2 + Ct^{-3}}$$

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(1) Bosqueje la línea fase de la Ecuación Diferencial:

$$\frac{dy}{dt} = \frac{-1}{y(y+3)},$$

y bosqueje la gráfica de la solución con condición inicial $y(0) = -\frac{3}{2}$ (Incluya valores negativos de t)

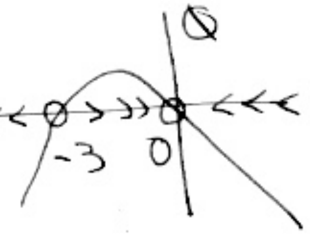
(2) Resuelva la Ecuación Diferencial:

$$(t^2+1)\frac{dy}{dt} + ty - t = 0$$

SOLUTION SET

(1) The Diff. Eq'n does not make any sense at $y=-3$, $y=0$

We have $\frac{dy}{dt} = \frac{1}{-(y^2+3y)}$. The denominator is a polynomial: $Q(y) = -(y^2+3y)$



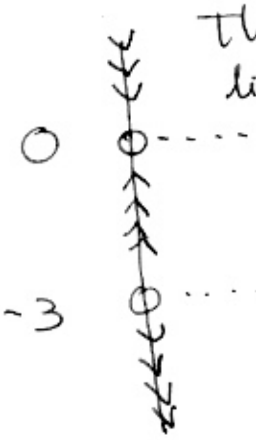
Hence $Q(y) > 0$, for $y \in (-3,0)$

$Q(y) < 0$, for $y \in (-\infty,-3) \cup (0,\infty)$

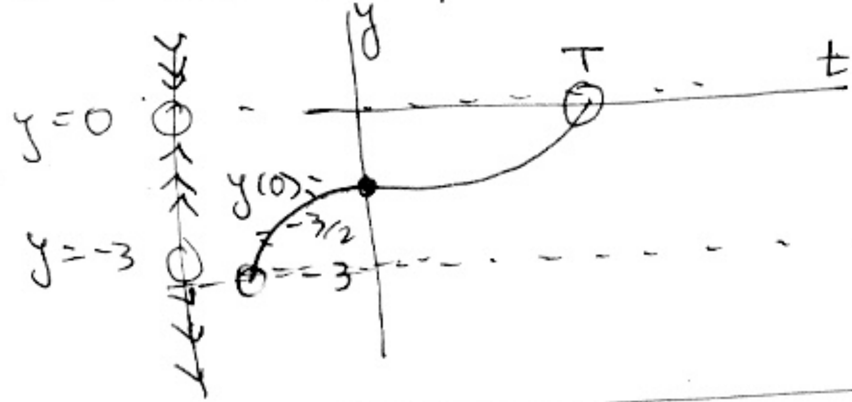
Then $\frac{dy}{dt} > 0$, for $y \in (-3,0)$

$\frac{dy}{dt} < 0$, for $y \in (0,\infty)$

The phase line is:



Using the phase line, we can sketch the graph of the initial value problem solution.



It never hits $y=0$, but there is a $T<\infty$.

$\lim_{t\to T} y(t) = 0$

(2) We write the Diff. Eq in standard form:

$$(t^2+1)\frac{dy}{dt} + ty = t \Rightarrow \frac{dy}{dt} + \frac{t}{1+t^2}y = \frac{t}{1+t^2}$$

The integrating factor is: $\mu(t) = e^{\int \frac{t}{1+t^2}dt} = e^{\frac{1}{2}\log(1+t^2)} =$

$$= \sqrt{1+t^2}.$$

And the integral is:

$$\int b(t)\mu(t)\,dt = \int \frac{t}{(1+t^2)}\sqrt{1+t^2}\,dt = \int \frac{t}{\sqrt{1+t^2}}\,dt = \sqrt{1+t^2}$$

Hence, the solution is $y(t) = \frac{1}{\mu(t)}\int \mu(t)\,b(t)\,dt =$

$$y(t) = \frac{1}{\sqrt{1+t^2}}\left(\sqrt{1+t^2}\right) + \frac{C}{\sqrt{1+t^2}}$$

$$\Rightarrow \boxed{y(t) = 1 + \frac{C}{\sqrt{1+t^2}}}$$