UAM - Azcapotzalco ECUACIONES DIFERENCIALES ORDINARIAS | Viernes, Febrero 23, 2018

Quiz #5 Nombre: ANSWER KEY

JUSTIFIQUE y ARGUMENTE sus respuestas. EXPLIQUE mientras desarrolla sus respuestas. Muestre sus cuentas

(1). Usando el método de la conjetura sensata, resuelva

$$\frac{dy}{dt} - 2y = 7e^{2t}$$

$$y(0) = 3$$

(2) Resuelva la Ec. Dif:

$$2xy^3 + 1 + \left(3x^2y^2 - \frac{1}{y}\right)\frac{dy}{dx} = 0$$

SOLUTION SET

(1)(a) We first solve the homogeneous equation:

$$\frac{dy_h}{dt} - 2y_h = 0.$$

Then: $\frac{dy_h}{dt} = 2y_h$; by Calculus, the solution is $y_h(t) = Ce^{2t}$.

(b) Now, the inhomogeneity $b(t) = 7e^{2t}$.

If we propose the particular solution $y_p(t) = \alpha e^{2t}$ will not work since it is the solution to the homogeneous equation. Then;

$$y_p(t) = \alpha t e^{2t} \quad \text{should work.}$$

We need to determine the coefficient α.

~~[illegible]~~

$$\frac{dy_p}{dt} - 2y_p = 7e^{2t}.$$

$$\frac{d}{dt}(\alpha t e^{2t}) - 2(\alpha t e^{2t}) = 7e^{2t}.$$

$$\alpha e^{2t} + 2\alpha t e^{2t} - 2\alpha t e^{2t} = 7e^{2t}$$

$$\alpha e^{2t} = 7e^{2t}. \Rightarrow \underline{\alpha = 7.}$$

Then, the solution is:

$$\boxed{y(t) = Ce^{2t} + 7te^{2t}}$$

Now $3 = y(0) = C + 0 \Rightarrow \boxed{C = 3}$

$$\boxed{y(t) = 3e^{2t} + 7te^{2t}}$$

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(2) We have $M(x,y) + N(x,y)\frac{dy}{dx} = 0$

with $M(x,y) = 2xy^3 + 1$; $N(x,y) = 3x^2y^2 - \frac{1}{y}$

Compute $\left.\begin{aligned} \frac{\partial M}{\partial y} &= 6xy^2 \\ \frac{\partial N}{\partial x} &= 6xy^2 \end{aligned}\right\}$ $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ then the equation is exact.

Integrating respect to x:

$$\left\{\begin{aligned} \frac{\partial \Psi}{\partial x} &= 2xy^3 + 1 \\ \frac{\partial \Psi}{\partial y} &= 3x^2y^2 - \frac{1}{y} \end{aligned}\right\} \Rightarrow \Psi(x,y) = x^2y^3 + x + h(y)$$

Now, $\frac{\partial \Psi}{\partial y} = 3x^2y^2 + h'(y)$. Comparin

Comparing with $\frac{\partial \Psi}{\partial y}$: $-\frac{1}{y} = h'(y) \Rightarrow h(y) = -\ln y$. Then.

$$\boxed{\Psi(x,y) = x^2y^3 + x - \ln y = C}$$