UAM-Azcapotzalco. ECUACIONES DIFERENCIALES | Viernes, Marzo 2, 2018.

Quiz #6 Nombre: ANSWER KEY.

JUSTIFIQUE y ARGUMENTE sus respuestas. EXPLIQUE mientras desarrolla sus respuestas. Muestre todas sus cuentas y también todo su desarrollo.

(1) Resuelva la Ec. Dif:

$$\frac{dy}{dt} = -\frac{y}{e^{t^2}} + \cos t.$$

(2) Determine si las siguientes funciones son linealmente independientes.

$$f(t) = \tan^2 t - \sec^2 t.$$

$$g(t) = 3$$

(3) Resuelva el siguiente problema de valores iniciales:

$$\ddot{y} - 4\dot{y} + 4y = 0$$

$$y(1) = 1$$

$$\dot{y}(1) = 1$$

SOLUTION SET:

(1) The Diff. Eq is linear, non-homogeneous, non-constant coeff's:

$$\frac{dy}{dt} = -\frac{y}{e^{t^2}} + \cos t.$$

We then, solve it by integrating factors: (since we cannot use the undetermined coeff., because of non-constant coeff)

$$\mu(t) = e^{-\int \frac{1}{e^{t^2}} dt} = e^{-\int e^{-t^2} dt}$$

We cannot solve $\int e^{-t^2} dt$ in terms of elementary functions; but it is a ~~definite~~ ~~rate~~ well-defined function:

$$\mu(t) = e^{-\int_{t_0}^{t} e^{-\tau^2} d\tau}.$$

Also:

$$\int \mu(t)\, b(t)\, dt = \int e^{-\int_{t_0}^{t} e^{-\tau^2} d\tau} \cos(t)\, dt$$

cannot be evaluated:

$$= \int_{t_0}^{t} e^{-\int_{t_0}^{\eta} e^{-\tau^2} d\tau} \cos(\eta)\, d\eta$$

Then, the solution is:

$$y(t) = \frac{1}{\mu(t)} \int_{t_0}^{t} \mu(t)\, b(t)\, dt + \frac{C}{\mu(t)}$$

$$\boxed{y(t) = \left(e^{\int_{t_0}^{t} e^{-\tau^2} d\tau}\right) \int_{t}^{t_0} e^{-\int_{t_0}^{\eta} e^{-\tau^2} d\tau} \cos(\eta)\, d\eta + C e^{\int_{t_0}^{t} e^{-\tau^2} d\tau}}$$

(2)(a) Notice that. | Problem (2). Three different ways to solve it

$$f(t) = \tan^2 t - \sec^2 t = \tan^2 t - (1+\tan^2 t)$$

$$= -1$$

Hence: $g(t) = -3 f(t)$, and they are linearly

independent.

(b) Also $W[f,g](t) = \det\begin{pmatrix} 1 & 3 \\ 0 & 0 \end{pmatrix} = 1\cdot 0 - 3\cdot 0 = 0,$

then, they are linearly independent.

(c) Also:

$$f'(t) = 2\tan t(\sec^2 t) - 2\sec t(\sec t \tan t)$$

$$= 2\tan t \sec^2 t - 2\sec^2 t \tan t$$

$$= 0.$$

Hence $W[f,g] = \det\begin{pmatrix} f & g \\ f' & g' \end{pmatrix} =$

$$= \det\begin{pmatrix} (\tan^2 t - \sec^2 t) & 3 \\ 0 & 0 \end{pmatrix}$$

$$= (\tan^2 t - \sec^2 t)\cdot 0 - 3\cdot 0$$

$$= 0$$

Then, they are linearly independent

③ Solve the IVP:

$$\ddot{y} - 4\dot{y} + 4y = 0; \quad \begin{array}{l} y(1) = 1 \\ \dot{y}(1) = 1 \end{array}$$

This is a linear, const. coeff's, homogeneous eq'n.
Look for solutions of the form: $y(t) = e^{rt}$. r = const.
Find r. Then, substitute into the equation:

$$r^2 - 4r + 4r = 0$$

$(r-2)^2 = 0$ We have repeated roots:
$r_1 = r_2 = 2$.

Then, the solutions are $y_1(t) = C_1 e^{2t} + C_2 t e^{2t}$
$y_2(t) = t e^{2t}$.

y and the solution to the homogeneous eq'n is:

$$y(t) = C_1 e^{2t} + C_2 t e^{2t}.$$

Now $\dot{y}(t) = 2C_1 e^{2t} + C_2 e^{2t} + 2C_2 t e^{2t}$.

Using the initial Conditions $e^2 C_1 + e^2 C_2 = y(1) = 1$.

$$2e^2 C_1 + 3e^2 C_2 = \dot{y}(1) = 1$$

$$e^2 \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \frac{1}{1 \cdot e^2} \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \frac{1}{e^2} \begin{pmatrix} 2 \\ -1 \end{pmatrix} \Rightarrow \boxed{y(t) = 2e^{2(t-1)} - t e^{2(t-1)}}$$

If the Initial Conditions would be:

$$y(0) = 1$$
$$\dot{y}(0) = 1$$

$$\left.\begin{aligned} C_1 + 0 &= y(0) = 1 \\ 2C_1 + C_2 + 0 &= \dot{y}(0) = 1 \end{aligned}\right\} \Rightarrow \begin{aligned} C_1 &= 1 \\ 2C_1 + C_2 &= 1 \end{aligned}$$

$$\Rightarrow \begin{aligned} C_1 &= 1 \\ C_2 &= 1 - 2C_1 = 1 - 2 = -1 \end{aligned} \qquad \boxed{\begin{aligned} C_1 &= 1 \\ C_2 &= -1 \end{aligned}}$$

Then: $\boxed{y(t) = e^{2t} - te^{2t}}$
