UAM-AZCAPOTZALCO INTRO AL CÁLCULO Viernes 9 de marzo de 2018

Quiz #7 Nombre: ANSWER KEY.

JUSTIFIQUE y ARGUMENTE sus respuestas. Muestre todos sus cuentas y desarrollo. EXPLIQUE mientras desarrolla sus respuestas.

(1) Calcule el siguiente límite:

$$\lim_{x \to 4} \frac{x^2 - 16}{|x-4|} :$$

(2) Calcule el límite:

$$\lim_{x \to 0} \frac{5}{\sqrt{5x+1} + 1}$$

(3) Encuentre las asíntotas de la función:

$$G(x) = \frac{2x^2 + 2x + 2}{x^2 - 16}.$$

ANSWER KEY

(1) We have to compute the side-limits:

When $x \to 4^+$, then $x-4>0$, then $|x-4| = x-4$.

Hence:

$$\lim_{x\to 4^+} \frac{x^2-16}{|x-4|} = \lim_{x\to 4^-} \frac{(x-4)(x+4)}{x-4} = \lim_{x\to 4^+} (x+4) = 8$$

Now, when $x \to 4^-$, then $x-4<0$, then $|x-4| = -(x-4)$.

Hence:

$$\lim_{x\to 4^-} \frac{x^2-16}{|x-4|} = \lim_{x\to 4^-} \frac{(x-4)(x+4)}{-(x-4)} = \lim_{x\to 4^-} \frac{x+4}{-1} = -8.$$

Since the side-limits are different,

the limit does NOT exist.

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(2) Then domain of $\left(\frac{5}{\sqrt{5x+1}+1}\right) = [-\frac{1}{5}, \infty)$, since $\sqrt{5x+1} \geq 0$ for $x \geq -\frac{1}{5}$, and $\sqrt{5x+1}+1 > 0$, for $x \in [-\frac{1}{5}, \infty)$.

In particular, $x=0 \in$ domain. and $\sqrt{5x+1}+1 = 2 \neq 0$.

Then, direct substitution applies:

$$\lim_{x\to 0} \frac{5}{\sqrt{5x+1}+1} = \frac{5}{\sqrt{1}+1} = \frac{5}{2}$$

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(3) Notice that $\text{Dom}(G) = \mathbb{R} \setminus \{4, -4\}$.

We have to compute the limits when $x \to 4$ and $x \to -4$.

$$\lim_{x\to 4} \frac{2x^2+2x+2}{x^2-16} = 2\left(\lim_{x\to 4} \frac{x^2+x+1}{x+4}\right)\left(\lim_{x\to 4} \frac{1}{x-4}\right)$$

$= 1 =$

$= 2 \cdot \frac{21}{8} \lim_{x \to 4} \frac{1}{x-4} = +\infty$, if $x \to 4^+$, and

it is $= -\infty$, if $x \to 4^{-1}$. In any case, this implies

the straight line $x = 4$ is a vertical asymptote.

Now,

$$\lim_{x \to -4} \frac{2x^2+2x+1}{x^2-16} = 2\left(\lim_{x \to -4} \frac{x^2+x+1}{x-4}\right)\left(\lim_{x \to -4} \frac{1}{x+4}\right)$$

$$= \left(2 \cdot \frac{21}{-8}\right) \lim_{x \to -4} \frac{1}{x+4} = +\infty, \text{ if } x \to -4^{-1},$$

and it is $-\infty$ if $x \to -4^+$. Hence, $x = -4$ is a

vertical asymptote.

Finally,

$$\lim_{x \to \infty} \frac{2x^2+2x+2}{x^2-16} = \lim_{x \to \infty} \frac{x^2\left(2+\frac{2}{x}+\frac{2}{x^2}\right)}{x^2\left(1-\frac{16}{x^2}\right)} = \lim_{x \to \infty} \frac{2+\frac{2}{x}+\frac{2}{x^2}}{1-\frac{16}{x^2}}$$

$$= \frac{2+0+0}{1-0} = 2.$$ Then, $y = 2$ is a horizontal asymptote.

Similarly,

$$\lim_{x \to -\infty} \frac{2x^2+2x+2}{x^2-16} = \lim_{x \to -\infty} \frac{2+\frac{2}{x}+\frac{2}{x^2}}{1-16/x^2} = 2.$$ Same horizontal asymptote.

Then

$$\boxed{\begin{array}{l} x = 4 \text{ and } x = -4 \text{ are vertical asymptotes} \\ y = 2 \text{ is a horizontal asymptote.} \end{array}}$$