UAM-Azcapotzalco | Ecuaciones Diferenciales Ordinarias | Viernes, Octubre 5, 2018

Quiz #2. Nombre: ANSWER KEY.

Instrucciones. Para obtener puntos, describa y explique su procedimiento y razonamiento. Muestre todas sus cuentas. Simplifique sus resultados. Escriba sus ideas con claridad y orden. Responda correctamente.

(1) Resuelva el problema de valores iniciales:

$$\frac{dy}{dt} - 4y + e^t = 0, \quad y(0) = 4/3.$$

(2) ¿Es la ecuación exacta? Si es así, resuélvala.

$$(2t + y) + (t - 2y)\frac{dy}{dt} = 0$$

(1) This is a first order, linear Diff. Eq. We use the integrating factor's method.

The integrating factor is: $\mu(t) = e^{-\int 4\,dt} = e^{-4t}$.

Now, $\int \mu(t) g(t)\,dt = \int e^{-4t}(-e^t)\,dt = -\int e^{-3t}\,dt$

$$= \frac{1}{3} e^{-3t}$$

The solution is:

$$y(t) = \frac{1}{\mu(t)} \int \mu(t) g(t)\,dt + \frac{C}{\mu(t)} = \frac{1}{e^{-4t}}\left(\frac{1}{3}e^{-3t}\right) + \frac{C}{e^{-4t}}.$$

$$\Rightarrow \boxed{y(t) = Ce^{4t} + \frac{1}{3}e^t}$$

Now: $\frac{4}{3} = y(0) = C + \frac{1}{3} \Rightarrow C = 1$

and so $\boxed{y(t) = e^{4t} + \frac{1}{3}e^t}$

$\Rightarrow 1 =$

② $(2t+y) + (t-2y)\frac{dy}{dt} = 0$.

We should check the cross-derivatives coincide:

$$\left.\begin{aligned}\frac{\partial}{\partial y}(2t+y) &= 1\\ \frac{\partial}{\partial t}(t-2y) &= 1\end{aligned}\right\} \text{ Then, it is exact!}$$

Find $\Psi(y,t)$, such that:
$$\frac{\partial \Psi}{\partial t} = 2t + y \quad \cdots\cdots \text{(I)}$$
$$\frac{\partial \Psi}{\partial y} = t - 2y \quad \cdots\cdots \text{(II)}$$

From eq (I): $\Psi(t,y) = t^2 + \cancel{t}y + g(y)$

$$\Rightarrow \frac{\partial \Psi}{\partial y} = t + g'(y). \text{ Compare with (II)}$$

$\Rightarrow g'(y) = -2y \Rightarrow g(y) = -y^2$.

Then, $\Psi(t,y) = t^2 + ty - y^2 + C_1$ and the implicit solution ~~is~~ $\Psi(t,y) = C$

becomes: $\boxed{t^2 + ty - y^2 = C}$

Solving for y:

$$y_{1,2}(t) = \frac{-t \pm \sqrt{t^2 + 4(t^2 - C)}}{-2}$$

Thus: the two explicit sol's are: $\boxed{y_{1,2}(t) = \frac{t \pm \sqrt{5t^2 + K}}{2}}$

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Instrucciones: Para obtener puntos, describa y explique su procedimiento y razonamiento. Muestre todas sus cuentas. Simplifique sus resultados. Escriba sus ideas con claridad y orden. Responda correctamente.

(1) Resuelva el problema de valores iniciales:

$$\frac{dy}{dt} - \frac{y}{t} = te^{t}, \quad y(1) = e-1$$

(2) ¿Es la siguiente ecuación exacta? Si es así, resuélvala.

$$(t^2-1)\frac{dy}{dt} + (2ty+3) = 0$$

(1) This is a 1^{st} order, linear Diff. Eq.

The integrating factor is $\mu(t) = e^{\int -\frac{1}{t}dt} = e^{-\log|t|} = \frac{1}{t}$.

Now, $\int \mu(t)g(t)dt = \int \frac{1}{t}(te^{t})dt = e^{t}$

The solution is:

$$y(t) = \frac{1}{\mu(t)}\int \mu(t)g(t)dt + \frac{c}{\mu(t)} = \frac{1}{1/t}e^{t} + \frac{c}{1/t} = t(e^{t}+C)$$

Now,

$$e-1 = y(1) = 1\cdot(e^{1}+C). \text{ Then } C=-1.$$

Hence, the solution is: $\boxed{y(t) = t(e^{t}-1)}$.

(2) The equation is exact.

$$\left.\begin{array}{l}\frac{\partial}{\partial t}(t^2-1) = 2t \\ \frac{\partial}{\partial y}(2ty+3) = 2t\end{array}\right\}, \text{ since they coincide.}$$

Then, there is a function $\Psi(t,y)$ such that:

$$\frac{\partial \Psi}{\partial t} = 2ty+3 \quad \text{------} \quad \text{(I)}$$

$$\frac{\partial \Psi}{\partial y} = t^2-1 \quad \text{------} \quad \text{(II)}$$

Hence, from (I) $\Psi(t,y) = t^2 y + 3t + g(y)$

$$\Rightarrow \frac{\partial \Psi}{\partial y} = t^2 + g'(y)$$

Comparing with (II): $g'(y) = -1 \Rightarrow g(y) = -y$.

Then, $\Psi(t,y) = t^2 y + 3t - y = (t^2-1)y + 3t$.

The implicit solution is $\Psi(t,y) = C$

i.e. $(t^2-1)y + 3t = C$

and the solution becomes:

$$\boxed{y(t) = \frac{C-3t}{t^2-1}}$$