UAM-Azcapotzalco | Ecuaciones Diferenciales Ordinarias | Viernes 12 de Octubre 2018

| Quiz #3 | Nombre: ANSWER KEY

Instrucciones: Para recibir puntos:

1) Responda correctamente
2) Describa y explique su procedimiento y razonamiento.
3) Muestre todas sus cuentas.
4) Simplifique sus resultados
5) Escriba sus datos con claridad y orden

(1) Resuelva la Ecuación Diferencial:

$$\frac{dy}{dt} = \frac{t^4 - t + y}{t}$$

(2) Resuelva la Ecuación:

$$\frac{dy}{dt} + ty^3 + \frac{y}{t} = 0$$

(1). The Diff Eq is linear: $\frac{dy}{dt} - \frac{1}{t}y = t^3 - 1$.

But it is not exact:

$$\underbrace{\left(-t^4 + t - y\right)}_{M(t,y)} + \underbrace{t}_{N(t,y)}\frac{dy}{dt} = 0.$$

We have: $\frac{\partial M}{\partial y} = -1$ and $\frac{\partial N}{\partial t} = 1$: Not exact.

Now, $\frac{d\mu}{dt} = \left(\frac{M_y - N_t}{N}\right)\mu \Rightarrow \frac{d\mu}{dt} = \left(\frac{-1-1}{t}\right)\mu$

=1 : It works!

$\frac{d\mu}{dt} = -\frac{2}{t}\mu \Rightarrow \boxed{\mu = t^{-2}}$ is the integrating factor:

Then, Diff Eq'n becomes:

$$t^{-2}(-t^4 + t - y) + t^{-2}\left(t \frac{dy}{dt}\right) = 0.$$

$$\left(-t^2 + \frac{1}{t} - \frac{y}{t^2}\right) + \frac{1}{t}\frac{dy}{dt} = 0$$

It is exact:

$$\left.\begin{aligned} \frac{\partial}{\partial y}\left(-t^2 + \frac{1}{t} - \frac{y}{t^2}\right) &= -\frac{1}{t^2} \\ \frac{\partial}{\partial t}\left(\frac{1}{t}\right) &= -\frac{1}{t^2} \end{aligned}\right\} \text{ Coincide.}$$

Then:

$$\frac{\partial F}{\partial t} = -t^2 + \frac{1}{t} - \frac{y}{t^2} \Rightarrow F(t,y) = -\frac{1}{3}t^3 + \operatorname{Log} t + \frac{y}{t} + g(y)$$

$$\Downarrow$$

$$\frac{\partial F}{\partial y} = \frac{1}{t} \qquad \frac{\partial F}{\partial y} = \frac{1}{t} + g'(y)$$

Comparing $\frac{\partial F}{\partial y}$: $g'(y) = 0$. $g(y) = \text{const}$

thus. $F(t,y) = -\frac{1}{3}t^3 + \operatorname{Log} t + \frac{y}{t}$

$$\Rightarrow -\frac{1}{3}t^3 + \log t + \frac{y}{t} = C$$

$$\Rightarrow \boxed{y(t) = Ct + \left(\frac{1}{3}t^4 - t\log t\right)}$$

is the solution to the Diff Eq'n

② The equation is of the Bernoulli type.

$$\frac{dy}{dt} + \frac{1}{t} y = -t y^3.$$

Under the transformation

$v(t) = y^{\alpha}(t)$ we have $\frac{dv}{dt} = \alpha y^{\alpha-1} \frac{dy}{dt} = \alpha y^{\alpha-1}\left(-\frac{1}{t}y - t y^3\right)$

$$= -\frac{\alpha}{t} y^{\alpha} - \alpha t y^{\alpha+2}$$

Take $\alpha + 2 = 0 \Rightarrow \boxed{\alpha = -2}$ y. $\boxed{v(t) = y^{-2}(t)}$

and:

$$\frac{dv}{dt} = \frac{2}{t} v + 2t,$$ which is linear;

$$\frac{dv}{dt} - \frac{2}{t} v = 2t.$$

The integrating factor is: $\mu(t) = e^{\int -\frac{2}{t} dt} = e^{-2\log t} = t^{-2}$

Now $\int \mu(t) q(t) dt = \int \frac{1}{t^2} \cdot 2t \, dt = 2 \int \frac{1}{t} dt = 2 \log t$.

thus:

$$v(t) = \frac{C}{t^{-2}} + \frac{1}{t^{-2}} (2 \log t) = t^2 (C + 2\log t)$$

Since $v = y^{-2}(t) \Rightarrow y(t) = \frac{1}{\sqrt{v(t)}}$ and so:

$$\boxed{y(t) = \frac{1}{t\sqrt{C + 2\log t}}}$$

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① Resuelva la ecuación.

$$\frac{dy}{dt} = \frac{2ty}{3t^2-y^2}$$

② Resuelva la Ecuación:

$$\frac{dy}{dt} - y = e^{2t}y^3$$

① We write the Diff Eq'n as:

$$\underbrace{-2ty}_{M(t,y)} + \underbrace{(3t^2-y^2)}_{N(t,y)}\frac{dy}{dt} = 0.$$

Now:

$$\left.\begin{aligned}\frac{\partial M}{\partial y} &= -2t. \\ \frac{\partial N}{\partial t} &= 6t\end{aligned}\right\}$$ It is different. Diff Eq'n not exact.

Now. $$\frac{d\mu}{dt} = \left(\frac{M_y - N_t}{N}\right)\mu \Rightarrow \frac{d\mu}{dt} = \left(\frac{-2t-6t}{3t^2-y^2}\right)\mu$$

$$\Rightarrow \frac{d\mu}{dt} = \left(\frac{-8t}{3t^2-y^2}\right)\mu$$

$-4=$

Depends on both
Does not work

Now, $\frac{d\mu}{dy} = \left(\frac{N_t - M_y}{M}\right)\mu \Rightarrow \frac{d\mu}{dy} = \left(\frac{6t + 2t}{-2ty}\right)\mu$

$\Rightarrow \frac{d\mu}{dy} = -\frac{4}{y}\mu$ It works!

$\Rightarrow \mu(y) = y^{-4}$

Then: $-2tyy^{-4} + y^{-4}(3t^2 - y^2)\frac{dy}{dt} = 0$

$-2ty^{-3} + (3t^2y^{-4} - y^{-2})\frac{dy}{dt} = 0$

$\partial_y(-2ty^{-3}) = 6ty^{-4}$
$\partial_t(3t^2y^{-4} - y^{-2}) = -6ty^{-4}$
Compatible. It is exact equation

Then $\frac{\partial F}{\partial t} = -2ty^{-3} \Rightarrow F = -t^2y^{-3} + g(y)$

$\frac{\partial F}{\partial y} = 3t^2y^{-4} - y^{-2} \Rightarrow \frac{\partial F}{\partial y} = +3t^2y^{-4} + g'(y)$

Comparing F_y: $g'(y) = -y^{-2} \Rightarrow g(y) = y^{-1}$

Then $F(t,y) = -t^2y^{-3} + y^{-1}$.

Then, the solution is: $\boxed{-t^2y^{-3} + y^{-1} = C}$

or $\boxed{-t^2 + y^2 = Cy^3}$

=S=

② It is a Bernoulli eq'n:

$$\frac{dy}{dt} - y = e^{2t} y^3.$$

Take $v = y^{\alpha}(t)$. Then $\frac{dv}{dt} = \alpha y^{\alpha-1}\left(\frac{dy}{dt}\right) =$

$$= \alpha y^{\alpha-1}\left(y + e^{2t} y^3\right) = \alpha y^{\alpha} + \alpha e^{2t} y^{\alpha+2}$$

Take $\alpha + 2 = 0$ Then $\alpha = -2$

$$\frac{dv}{dt} = -2v - 2e^{2t}$$ linear eq'n.

Integrating factor: $\mu(t) = e^{\int 2 dt} = e^{2t}$.

and $\int \mu(t) g(t)\, dt = \int e^{2t}(-2e^{2t})\,dt = -2\int e^{4t}\,dt$

$$= -\frac{2}{4} e^{4t} = -\frac{1}{2} e^{4t}.$$

Hence: $v(t) = \frac{1}{e^{2t}}\left(-\frac{1}{2} e^{4t}\right) + \frac{C}{e^{2t}} = -e^{2t} + \frac{C}{e^{2t}}$

$v = y^{-2}(t)$

Then $y(t) = \frac{1}{\sqrt{v(t)}}$ $\Rightarrow$ $\boxed{y(t) = \frac{1}{Ce^{-2t} - e^{2t}}}$

$= 6 =$