**ECUACIONES DIFERENCIALES ORDINARIAS**
**EXAMEN GLOBAL. TRIMESTRE 19-I**
**Turno vespertino**

Nombre: ANSWER KEY Matrícula: ______

**Indicación:** El examen global comprende los ejercicios marcados con ♣ y entre paréntesis aparece el valor de cada uno de ellos.
Si presentas un parcial, el examen comprende los ejercicios de ese parcial.
Los resultados deben mostrar el procedimiento.

## Primera parte

En los ejercicios 1, 2 y 3 resuelve la ecuación diferencial

1♣**(1.0)**. $y' + y = yxe^{x+2}$
2♣**(1.0)**. $(6x^2y^2 - 4y^4 + 2\sec x^4)dx + (2x^3y - 4xy^3)dy = 0$
3♣**(1.0)**. $y' + y = xy^3$

4. La población de una comunidad se incrementa a una razón proporcional al numero de habitantes presentes al tiempo $t$; si en 5 años se duplica la población inicial $P_0$
a) encuentra el número de habitantes en la comunidad al tiempo $t$
b) ¿en cuánto tiempo se triplicará la población?
c) si a los 3 años hay 10,000 habitantes, ¿cuál era la población inicial $P_0$?

5♣**(1.0)**. Un tanque contiene 320 litros de agua pura. Se vierte al tanque, a razón de 4 $l/min$, una solución salina que contiene 5 $kg/l$ de sal y la mezcla homogénea sale del recipiente a razón de 2 $l/min$
a) determina la cantidad de sal a cualquier instante $t$
b) ¿cuál es la concentración de sal cuando el volumen es de 480 litros?

## Segunda parte

1♣**(1.0)**. Encuentra la solución de la ecuación diferencial

$$y'' + (\tan x - 2\cot x)y' + 2\cot^2 x\; y = 0$$

sabiendo que $y_1 = \operatorname{sen} x$ es una solución.

2♣(**1.5**). Resuelve la ecuación diferencial usando el método de los coeficientes indeterminados

$$y'' + 3y' + 2y = -6e^{-x} + 2x$$

3♣(**1.5**). Resuelve la ecuación diferencial

$$y'' - 6y' + 9y = e^{3x} \ln x$$

4. Resuelve el problema de valor inicial
$y'' + y' = 6x + 2\cos x$
$y(0) = 1$
$y'(0) = 1$

**Tercera parte**

1♣(**2.0**). Una masa de 1 *slug* está suspendida de un resorte cuya constante es 9 $lb/pie$. Inicialmente la masa se pone en movimiento desde un punto que está 1 *pie* sobre la posición de equilibrio y con una velocidad dirigida hacia arriba de $\sqrt{3}$ $pie/seg$
a) determina las ecuaciones de movimiento y de la velocidad, como funciones del tiempo
b) escribe la ecuación de movimiento en la forma $x(t) = A\,\mathrm{sen}(wt + \phi)$
c) ¿en qué instante pasa la masa por la posición de equilibrio en dirección hacia abajo por tercera vez?

2. Una masa que pesa 4 *lb* se sujeta a un resorte de constante igual a 2 $lb/pie$; el sistema se sumerge en un medio que ofrece una fuerza de resistencia numéricamente igual a la velocidad instantánea. En el tiempo $t = 0$, la masa se suelta con una velocidad de 8 $pie/seg$ en dirección hacia abajo desde una punto que está 1 pie arriba de la posición de equilibrio, determina:
a) la ecuación movimiento
b) el instante en que pasa por la posición de equilibrio

3. Un resorte es alargado 1.5 pulgadas por una masa cuyo peso es de 16 *lb*. Si esta masa se suelta 4 pulgadas abajo de la posición de equilibrio con una velocidad dirigida hacia abajo de 4 $pie/seg$ y se aplica al sistema una fuerza externa dada por $F(t) = 360\cos 4t$, determina la ecuación de movimiento y explica si el sistema está en resonancia.

UAM - AZCAPOTZALCO

Martes, Julio 23, 2019.

ECUACIONES DIFERENCIALES ORDINARIAS. Examen Global ANSWER KEY

ANSWER KEY PARTE I

(1) $y' + y = y\,x\,e^{x+2}$. This is a separable eq'n.

Rewriting the equation as: $\frac{y'}{y} = (x e^{x+2} - 1)$

we get $\frac{d}{dx}(\log|y|) = (x e^{x+2} - 1) \Rightarrow \log|y| = \int (x e^{x+2} - 1)dx$

$$\Rightarrow \log|y| = e^2 \int x e^x dx - x$$

Integrating by parts:

$$\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C_1 = (x-1)e^x + C_1.$$

Hence: $\log y = (x-1)e^{x+2} + e^2 C_1 - x$.

$$\boxed{y(x) = C e^{(x-1)e^{x+2} - x}}$$

(2) This is a nonlinear Diff Eq. of first order. We must find a generalized integrating factor:

$$\underbrace{\left[6x^2y^2 - 4y^4 + 2\sec(x^4)\right]}_{M(x,y)} + \underbrace{\left(2x^3y - 4xy^3\right)}_{N(x,y)} \frac{dy}{dx} = 0$$

Is this an exact equation? No.

$$\left.\begin{aligned} \frac{\partial M}{\partial y} &= 12x^2y - 16y^3 \\ \frac{\partial N}{\partial x} &= 6x^2y - 4y^3 \end{aligned}\right\} \text{Not the same.}$$

To be exact:

$$(\mu M) + (\mu N)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{\partial}{\partial y}(\mu M) = \frac{\partial}{\partial x}(\mu N)$$

Assuming $\mu(x,y)=\mu(x)$: $\quad \mu M_y = \mu_x N + \mu N_x$.

$\Rightarrow \dfrac{\mu_x}{\mu} = \dfrac{M_y - N_x}{N}$ should depend on $x$ only.

$$\frac{1}{\mu}\frac{d\mu}{dx} = \frac{M_y - N_x}{N} = \frac{(12x^2y - 16y^3) - (6x^2y - 4y^3)}{2x^3y - 4xy^3} = \frac{6x^2y - 12y^3}{2xy(x^2-2y^2)}$$

$$= \frac{6y(x^2-2y^2)}{2xy(x^2-2y^2)} = \frac{3}{x}.$$ It works: $\mu(x)$ is an integrating factor

$\dfrac{1}{\mu}\dfrac{d\mu}{dx} = \dfrac{3}{x} \Rightarrow \log\mu = 3\log x \Rightarrow \mu(x) = x^3$ is the integrating factor

Multiply eq'n by $x^3$:

$$\left[6x^5y^2 - 4x^3y^4 + 2x^3\sec(x^4)\right] + \left[2x^6y - 4x^4y^3\right]\frac{dy}{dx} = 0$$

There should be a function $F(x,y)$ such that:

$$\frac{\partial F}{\partial x} = 6x^5y^2 - 4x^3y^4 + 2x^3\sec(x^4) \quad \ldots\ldots\ldots (*)$$

$$\frac{\partial F}{\partial y} = 2x^6y - 4x^4y^3 \quad \ldots\ldots\ldots (**)$$

Integrate $(*)$ w.r.t. $x$: $F(x,y) = x^6y^2 - x^4y^4 + 2\int x^3\sec(x^4)dx + g(y)$

for some function $g(y)$ to be determined. Now:

$$\int x^3\sec(x^4)dx = \underbrace{\frac{1}{4}\int \sec(t)dt}_{t=x^4} = \frac{1}{4}\operatorname{Log}|\sec t + \tan t|.$$ Thus:

$$F(x,y) = x^6y^2 - x^4y^4 + \frac{1}{2}\operatorname{Log}|\sec(x^4) + \tan(x^4)| + g(y) \quad \ldots\ldots (***)$$

Computing derivative w.r.t $y$:

$\partial_y F = 2x^6y - 4x^4y^3 + 0 + g'(y)$, and cf. $(**) \Rightarrow g'(y) = 0$

$\Rightarrow g(y) = $ Const. Then $\boxed{F(x,y) = x^6y^2 - x^4y^4 + \frac{1}{2}\operatorname{Log}|\sec x^4 + \tan x^4| = K}$

is the ~~implicit~~ solution.

(3) $y' + y = xy^3$ : Bernoulli-type eq'n

Multiply by $y^{-3}$ : $y^{-3}y' + y^{-2} = x$. Define $v(x) \equiv y^{-2}$.

Then: $\frac{dv}{dx} = -2y^{-3}\frac{dy}{dx} = -2(-y^{-2} + x) = 2y^{-2} - 2x = 2v - 2x$

$\frac{dv}{dx} - 2v = -2x$. We know how to solve this eq'n.

The integrating factor is $\mu(x) = e^{-2x} \Rightarrow e^{-2x}\frac{dv}{dx} - 2e^{-2x}v = -2e^{-2x}x$

$\Rightarrow \frac{d}{dx}(e^{-2x}v) = -2e^{-2x}x. \Rightarrow e^{-2x}v(x) = -2\int xe^{-2x}dx + C$

$\Rightarrow e^{-2x}v(x) = -2\left[\frac{xe^{-2x}}{-2} + \frac{1}{2}\int e^{-2x}dx\right] = xe^{-2x} + \frac{1}{2}e^{-2x} + C$

$\Rightarrow v(x) = x + \frac{1}{2} + Ce^{2x} \Rightarrow \boxed{y(x) = (v(x))^{-1/2} = \frac{1}{\sqrt{x + \frac{1}{2} + Ce^{2x}}}}.$

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(4) This population follows Malthus' Law: $\frac{dP}{dt} = \alpha P$,

$P(t) = P(0)e^{\alpha t}$. We know that $P(5) = 2P(0) \Rightarrow P(0)e^{5\alpha} = 2P(0)$

$\Rightarrow \boxed{\alpha = \frac{1}{5}\operatorname{Log} 2}$ 1/year. is the birth-rate. Hence: $P(t) = P(0)\exp(\alpha t)$

$(\approx 0.138$ 1/year).

$= P(0)\exp\left(\frac{t}{5}\operatorname{Log}2\right) = P(0)\exp\left(\operatorname{Log}(2^{t/5})\right) \Rightarrow \boxed{P(t) = P(0)2^{t/5}}$

inhabitants at time t

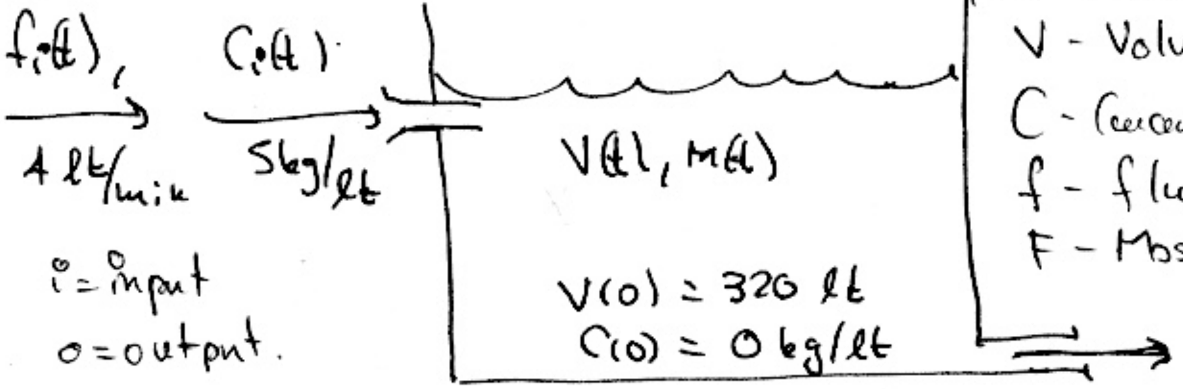
(b) We want to know T, s.t. $P(T) = 3P(0) \Rightarrow P(0)2^{T/5} = 3P(0)$

$\Rightarrow \boxed{T = 5\frac{\log 3}{\log 2}}$ years $\approx 7.92$ years

(c) We know that $P(3) = 10^4$ inhabitants: $P(0)2^{3/5} = 10^4$.

$\Rightarrow \boxed{P(0) = \frac{10^4}{2^{3/5}} \approx 6,597}$ inhabitants

(5)




M - Mass (salt) [kg]
V - Volume (water) [lt]
C - Concentration (salt)
f - flux [lt/min].
F - Mass flux [kg/min]

i = input
o = output.

$C(t) = \frac{M(t)}{V(t)} \left[\frac{kg}{lt}\right]$ Concentration of salt at time t.

$\frac{dV}{dt} = f_i(t) - f_o(t) = 2\ lt/min \Rightarrow V(t) = 2t + V(0)\ lt.$

$C_o(t) = C(t)$.

$\frac{dM}{dt} = F_i(t) - F_o(t) = f_i C_i - f_o C_o(t) \left[\frac{lt}{min} \cdot \frac{kg}{lt}\right] = 20 - 2\,C(t)$

$\Rightarrow \frac{d}{dt}(VC) = 20 - 2C(t) \Rightarrow \frac{dC}{dt} = \frac{20 - 4C(t)}{V(t)} = \frac{20 - 4C(t)}{2t + V(0)}.\ = \frac{20-4C}{V(t)}$

The integrating factor is: $\mu(t) = e^{\int \frac{4}{2t+V(0)} dt} = e^{2\log|2t+V(0)|}$

$\mu(t) = (2t + V(0))^2 = V^2(t) \Rightarrow V^2\left(\frac{dC}{dt} + \frac{4C}{V(t)}\right) = V^2\left(\frac{20}{V(t)}\right)$

$\Rightarrow \frac{d}{dt}(V^2 C(t)) = 20\,V(t) \Rightarrow V^2(t)C(t) = 20\int 2t + V(0)\,dt = 20(t^2 + V(0)t).$

$\Rightarrow \boxed{C(t) = \frac{20\,(t^2 + V(0)t)}{(2t + V(0))^2}}$ is the concentration at time t.
$(V(0) = 320\ lt)$

Since: $M(t) = V(t)C(t) = \frac{20(t^2 + V(0)t)}{V(t)} \Rightarrow \boxed{M(t) = \frac{20(t^2 + V(0)t)}{2t + V(0)}}$.

(b) When $V(t) = 480\ lt \Rightarrow 2t + 320 = 480 \Rightarrow \boxed{t = 80 \text{ seg}}$

$\boxed{C(80) = \frac{20\,(80^2 + V(0)\,80)}{(160 + V(0))^2} = \frac{1}{12}\ kg/lt}$

PARTE II

(1) $y'' + (\tan x - 2\cot x)y' + (2\cot^2 x)y = 0.$

Let us check that: $y_1(x) = \sin x$ is solution:

$$-\sin x + (\tan x - 2\cot x)\cos x + (2\cot^2 x)\sin x = 0$$

$$-\sin x + \left(\sin x - 2\frac{\cos^2 x}{\sin x}\right) + 2\frac{\cos^2 x}{\sin x} = 0 \checkmark$$

It works. Now, define $y_2(x) = v(x)y_1(x)$: $v(x)$ is unknown.

and we assume $y_2'' + p(x)y_2' + q(x)y_2 = 0.$

Then $(vy_1)'' + p(vy_1)' + q(vy_1) = 0$:

$$y_1 v'' + 2v'y_1' + vy_1'' + pv'y_1 + pvy_1' + qvy_1 = 0$$

$$y_1 v'' + (2y_1')v' + (py_1)v' + \overbrace{(y_1'' + py_1' + qy_1)}^{=0} v = 0$$

$$\Rightarrow y_1 \frac{d^2v}{dx^2} + (2y_1' + py_1)\frac{dv}{dx} = 0.$$

i.e. $$\sin x\frac{d^2v}{dx^2} + \left(2\cos x + (\tan x - 2\cot x)\sin x\right)\frac{dv}{dx} = 0$$

$$\sin x\frac{d^2v}{dx^2} + \left(2\cos x + \frac{\sin^2 x}{\cos x} - 2\cos x\right)\frac{dv}{dx} = 0$$

$$\sin x\frac{d^2v}{dx^2} + \frac{\sin^2 x}{\cos x}\frac{dv}{dx} = 0 \Rightarrow \frac{d^2v}{dx^2} = -\frac{\sin x}{\cos x}\frac{dv}{dx}$$

$$\Rightarrow \frac{v''}{v'} = -\frac{\sin x}{\cos x} \Rightarrow \operatorname{Log}|v'| = \operatorname{Log}|\cos x| \Rightarrow v' = \cos x$$

$$\Rightarrow v(x) = \sin x \Rightarrow \boxed{y_2(x) = \sin^2 x}$$

$$\Rightarrow y_2(x) = v(x)\sin x. \Rightarrow$$

$$\boxed{y(x) = C_1\sin x + C_2\sin^2 x}$$ is the sol'n to the Diff. Eq'n.

② Solving the homogeneous eq'n: $y_h'' + 3y_h' + 2y_h = 0$.

we get $(r+2)(r+1) = 0 \Rightarrow y_h(x) = C_1 e^{-x} + C_2 e^{-2x}$.

Now, propose: $y_p(x) = Ax e^{-x} + Bx + C$

(Since $Ae^{-x}$ repeats $C_1 e^{-x}$ in $y_h(x)$).

$y_p'(x) = A(1-x)e^{-x} + B$ Hence,

$y_p''(x) = -A(2-x)e^{-x}$.

$$-A(2-x)e^{-x} + \left[3A(1-x)e^{-x} + 3B\right] + 2Axe^{-x} + 2Bx + 2C =$$
$$= -6e^{-x} + 2x.$$

$$e^{-x}\left[-2A + Ax + 3A - 3Ax + 2Ax\right] + \left[2Bx + 3B + 2C\right] = -6e^{-x} + 2x.$$

$$\Rightarrow Ae^{-x} + 2Bx + (3B+2C) = -6e^{-x} + 2x.$$

$\Rightarrow \boxed{A = -6}$ $\boxed{B = 1}$ $3B + 2C = 0 \Rightarrow \boxed{C = -\frac{3}{2}B = -\frac{3}{2}}$

Then, the solution to the Diff. Eq'n is:

$$\boxed{y(x) = C_1 e^{-x} + C_2 e^{-2x} - 6xe^{-x} + x - \frac{3}{2}.}$$

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③ We have to solve the homogeneous eq'n:

$$y_h'' - 6y_h' + 9y_h = 0$$

We get $(r-3)^2 = 0 \Rightarrow y_h(x) = C_1 e^{3x} + C_2 x e^{3x} = C_1 y_1$
$= C_1 y_1(x) + C_2 y_2(x)$

Use Variation of parameters:

$$W[y_1, y_2](x) = \det\begin{pmatrix} e^{3x} & x e^{3x} \\ 3e^{3x} & e^{3x}(1-3x) \end{pmatrix} = e^{6x}(1-3x) - 3xe^{6x} = e^{6x} \neq 0$$

Now: $A(x) = -\int \frac{f(x)\, y_2(x)}{a(x)\, W[y_1, y_2]}\, dx = -\int \frac{e^{3x}(\log x)\, x e^{3x}}{1\cdot e^{6x}}\, dx$

$= -\int x\log x\, dx = -\frac{1}{2}x^2\log x + \frac{1}{2}\int x^2 \frac{1}{x}\, dx = -\frac{1}{2}x^2\log x + \frac{1}{4}x^2$

$= \frac{1}{4}x^2(1 - 2\log x)$

Similarly

$B(x) = \int \frac{f(x)\, y_1(x)}{a(x)\, W[y_1, y_2]}\, dx = \int \frac{e^{3x}(\log x) e^{3x}}{1\cdot e^{6x}}\, dx = \int \log x\, dx$

$= x(\log x - 1)$

Thene, the solution: $y(x) = (C_1 + A(x))y_1(x) + (C_2 + B(x))y_2(x)$

Becomes: $y(x) = \left(C_1 + \frac{1}{4}x^2(1 - \log x)\right)e^{3x} + \left(C_2 + x(\log x - 1)\right)x e^{3x}$

$$\boxed{y(x) = \left(C_1 + xC_2 + \frac{1}{4}x^2 - x + \frac{3}{4}x^2\log x\right) e^{3x}}$$

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(4) $y'' + y' = 6x + 2\cos x \Rightarrow y' + y = 3x^2 + 2\sin x + C_1$

The sol'n to the homogenous eq'n is: $y_h(x) = C_2 e^{-x}$

A particular solution has the form:

$$y_p(x) = Ax^2 + Bx + C + \alpha\cos x + \beta\sin x.$$

$$y_p'(x) = 2Ax + B - \alpha\sin x + \beta\cos x.$$

$\Rightarrow$

$$Ax^2 + (2A + B)x + B + C + (\alpha + \beta)\cos x + (\beta - \alpha)\sin x = $$

$$= 3x^2 + 2\sin x + C_1.$$

$\Rightarrow \boxed{A = 3}$; $2A + B = 0 \Rightarrow \boxed{B = -6}$; $\boxed{C = C_1}$ $\quad \begin{array}{l}\alpha + \beta = 0 \\ \beta - \alpha = 2\end{array}$ $\boxed{\begin{array}{l}\alpha = -1 \\ \beta = 1\end{array}}$

$$\Rightarrow \boxed{y(x) = C_2 e^{-x} + 3x^2 - 6x + C_1 - \cos x + \sin x}$$

Now:

$$y(x) = C_2 e^{-x} + 3x^2 - 6x + C_1 - \cos x + \sin x.$$

$$y'(x) = -C_2 e^{-x} + 6x - 6 + \sin x + \cos x$$

$$\left.\begin{array}{l} y(0) = C_2 - 1 + C_1 = 1 \\ y'(0) = -C_2 - 6 + 1 = 1 \end{array}\right\} \quad \begin{array}{l} C_1 + C_2 = 2 \\ -C_2 = 6 \end{array} \Rightarrow \boxed{\begin{array}{l} C_1 = 8 \\ C_2 = -6 \end{array}}$$

$$\boxed{y(x) = -6e^{-x} + 3x^2 - 6x + 8 - \cos x + \sin x}$$

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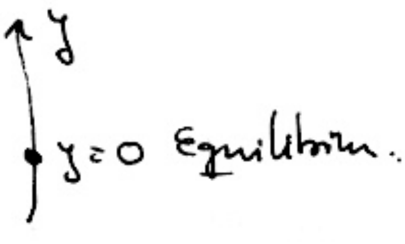
TERCERA PARTE:




(1) $m = 1$ slug ; $y(0) = 1$ ft
$k = 9$ lb/ft ; $\dot{y}(0) = \sqrt{3}$ ft/sec

The eq'n of motion: $\ddot{y} + 9y = 0$ ; $y(t) = C_1\cos(3t) + C_2\sin(3t)$
$\dot{y}(t) = -3C_1\sin(3t) + 3C_2\cos(3t)$

$$\Rightarrow \begin{array}{l} y(0) = C_1 = 1\text{ft} \\ \dot{y}(0) = 3C_2 = \sqrt{3} \end{array} \quad \begin{array}{l} C_1 = 1 \text{ ft} \\ C_2 = \frac{\sqrt{3}}{3} \text{ ft} \end{array} . \quad \boxed{y(t) = \cos(3t) + \frac{1}{\sqrt{3}}\sin(3t).}$$

(b) $\omega = 3(1/\text{sec.})$ frequency: $\boxed{A = \sqrt{1^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2}{\sqrt{3}} \text{ ft}}$

Since $C_1 > 0 \Rightarrow \varphi = \text{Arctan}\left(\sqrt{3}\right) = \frac{\pi}{3} + 0.$
and $\varphi = \text{Arctan}\left(\frac{C_1}{C_2}\right) \Rightarrow$

$$\Rightarrow \quad y(t) = \frac{2}{\sqrt{3}}\cos\left(3t - \frac{\pi}{3}\right)$$

But
$\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$
$\theta = 3t - \frac{\pi}{3}$
$\theta + \frac{\pi}{2} = 3t - \frac{\pi}{3} + \frac{\pi}{2} =$
$= 3t - \frac{\pi}{6}.$

$$\Rightarrow \boxed{y(t) = \frac{2}{\sqrt{3}}\sin\left(3t - \frac{\pi}{6}\right)}$$

(c) $3t - \frac{\pi}{6} = 5\pi \Rightarrow \boxed{t = \frac{31}{18}\pi \text{ seg}}$

(2) $F = 4\,lb$, but $F = mg \Rightarrow m = \frac{F}{g} = \frac{4}{32}\,\frac{lb}{ft/sec.}$

$\Rightarrow \underbrace{m = \frac{1}{8}\text{ slug}}_{\text{is the mass}} \quad \underbrace{k = 2\,lb/ft}_{\text{is Hooke's const}} \quad \underbrace{b = 1\,lb.sec/ft}_{\text{Friction const.}}$.

$y$

$y=0$ Equilibrium

$y(0) = 1\,ft$

$\dot{y}(0) = -8\,ft/sec.$

Eq'n of motion:

$$\boxed{\frac{1}{8}\ddot{y} + \dot{y} + 2 = 0}$$

Characteristic eq'n: $r^2 + 8r + 16 = 0$

$(r+4)^2 = 0$ Critically damped.

$$\boxed{y(t) = (C_1 t + C_2)e^{-4t}}.$$

At $\quad \dot{y}(t) = (C_1 - 4C_1 t - 4C_2)e^{-4t}.$

At $= t=0 \quad y(0) = C_2 = 1\,ft$

$\dot{y}(0) = (C_1 - 4C_2) = -8\,ft/sec$

$\Rightarrow \boxed{\begin{array}{l} C_1 = -4\frac{ft.}{sec} \\ C_2 = 1\,ft \end{array}}$

$$\boxed{y(t) = (-4t+1)e^{-4t}}$$

It passes through the origin; when $\boxed{t = \frac{1}{4}\text{ sec}}$.

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(3) $12\,in = 1\,ft$ ; $1\,in = \frac{1}{12}\,ft$ ; $1.5\,in = \frac{3}{2}\,in = \frac{3}{2\cdot 12}\,ft = \frac{1}{8}\,ft$

$m = 16\,lb = 16\left(\frac{1}{32}\text{ slug}\right) = \frac{1}{2}$ slug.

Then, $k = \frac{F}{\ell} = \frac{16\,lb}{\frac{1}{8}\,ft} = 128\,lb/ft.$

$F_{ext}(t) = 360\cos(4t) \Rightarrow$ Eq'n of motion:

$$\frac{1}{2}\ddot{y} + 128\,y = 360\cos(4t).$$

2) $\ddot{y} + 256y = 720\cos 4t.$

$\omega^2 = 256 \frac{1}{sec^2} = 2^8 \, 1/sec \quad \boxed{\omega = 2^4 \, 1/sec} = 16/sec.$

External frequency $= 4 \, 1/sec$. $\boxed{\text{There is no resonance}}$