UAM - AZCAPOTZALCO | ECUACIONES DIFERENCIALES | Vie. Sept 13, 2019.

Quiz #1 Nombre: ANSWER KEY

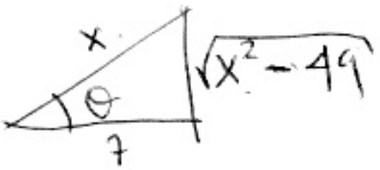
1. Calcule $\int \frac{\sqrt{x^2-49}}{x}\,dx$, para $x > 7$.

2. Calcule $\int x \sin\frac{x}{2}\,dx$

3. Usando la definición, calcule la derivada de $\frac{1}{x}$.

(1.) Sea $x = 7\sec\theta$ $\quad \frac{dx}{d\theta} = 7\sec\theta\tan\theta$

$$x^2 - 49 = 49(\sec^2\theta - 1) = 49\tan^2\theta$$



$\cos\theta = \frac{1}{\sec\theta} = \frac{7}{x}$

Por lo tanto:

$$\int \frac{\sqrt{x^2-49}}{x}\,dx = \int \frac{7\tan\theta}{7\sec\theta}\, 7\sec\theta\tan\theta\, d\theta$$

$$= 7\int \tan^2\theta\, d\theta = \int (1+\tan^2\theta) - (1)\, d\theta$$

$$= \int \frac{d}{d\theta}(\tan\theta)'\, d\theta - \theta = \tan\theta - \theta + C$$

$$= \boxed{\frac{\sqrt{x^2-49}}{7} - \mathrm{Arccos}\left(\frac{7}{x}\right) + C}$$

(2) $$\int x \sin\frac{x}{2}\,dx = 2\cdot 2\int y \sin y\,dy = 4\left[y(-\cos y) + \int \cos y\right]$$

$y = \frac{x}{2}$ Partes

$$= 4\left[-y\cos y + \sin y + C\right]$$

$$\boxed{\int x \sin\frac{x}{2}\,dx = -2x\cos\frac{x}{2} + 4\sin\frac{x}{2} + C}$$

(3) Por definição

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \lim_{h\to 0}\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0}\frac{\left(\frac{x-(x+h)}{(x+h)(x)}\right)}{h} =$$

$$= \lim_{h\to 0}\frac{\left(\frac{(-h)}{(x+h)x}\right)}{h} = \lim_{h\to 0}\frac{-h}{(x+h)x h} =$$

$$= \lim_{h\to 0}\frac{-1}{(x+h)x} = \frac{-1}{x^2}$$

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Quiz #1 Nombre:

1. Calcule $\int \frac{dx}{x^2\sqrt{x^2-1}}$, para $x > 1$

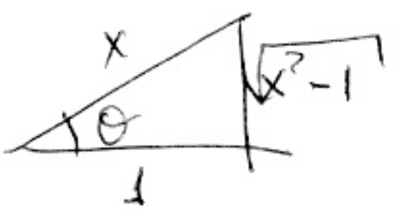
2. Calcule $\int x \cdot \operatorname{Log} x \, dx$ (Log es ln)

3. Usando la definición, calcule la derivada de $\sqrt{x}$

1. Sea $x = \sec\theta$ $\quad \frac{dx}{d\theta} = \sec\theta \tan\theta$

y $x^2 - 1 = \sec^2\theta - 1 = \tan^2\theta$

Entonces:



$$\int \frac{1}{x^2\sqrt{x^2-1}}\, dx = \int \frac{\sec\theta \tan\theta \, d\theta}{\sec^2\theta \tan\theta}$$

$$= \int \frac{1}{\sec\theta} \, d\theta$$

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{x}$

$$= \int \cos\theta \, d\theta = \sin\theta + C$$

$$\Rightarrow \boxed{\int \frac{1}{x^2\sqrt{x^2-1}}\, dx = \frac{\sqrt{x^2-1}}{x} + C'}$$

(2) $$\int x \log x \, dx \underset{\text{Partes}}{=} \frac{x^2}{2} \log x - \int \frac{x^2}{2} \frac{1}{x} \, dx$$

$$= \frac{x^2}{2} \operatorname{Log} x - \int \frac{x}{2} \, dx = \frac{x^2}{2} \log x - \frac{x^2}{4} + C$$

$$\Rightarrow \boxed{\int x \log x \, dx = \frac{x^2}{2} \left(\log x - 2 \right) + C}$$

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(3) Por la definición:

$$\frac{d}{dx}(\sqrt{x}) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$
