UAM-Azcapotzalco Ecuaciones Diferenciales Ordinarias | V-20/Sept/2019

Quiz #2. Nombre: Answer Key

(1) La función $y(t) = t^2$, ¿es solución de la Ec. Diferencial?

$$t^2\ddot{y} + \frac{1}{2}t\dot{y} - y = t^2$$

(2) Para la ecuación diferencial, encuentre los valores de $y(t)$ para los cuales (a) $y(t)$ está en equilibrio, (b) $y(t)$ es creciente y (c) $y(t)$ es decreciente.

$$\frac{dy}{dt} = 4y^3 + 2y^2 - 2y$$

(1) We should compute $\dot{y}$ and $\ddot{y}$, and substitute into the equation to check it is solution or not:

$y(t) = t^2$, $\dot{y}(t) = 2t$, $\ddot{y} = 2$

$$t^2\ddot{y} + \frac{1}{2}t\dot{y} - y = t^2 \cdot 2 + \frac{1}{2}t \cdot 2t - t^2 = 2t^2 + t^2 - t^2$$

$= 2t^2 \neq t^2$. It is not solution

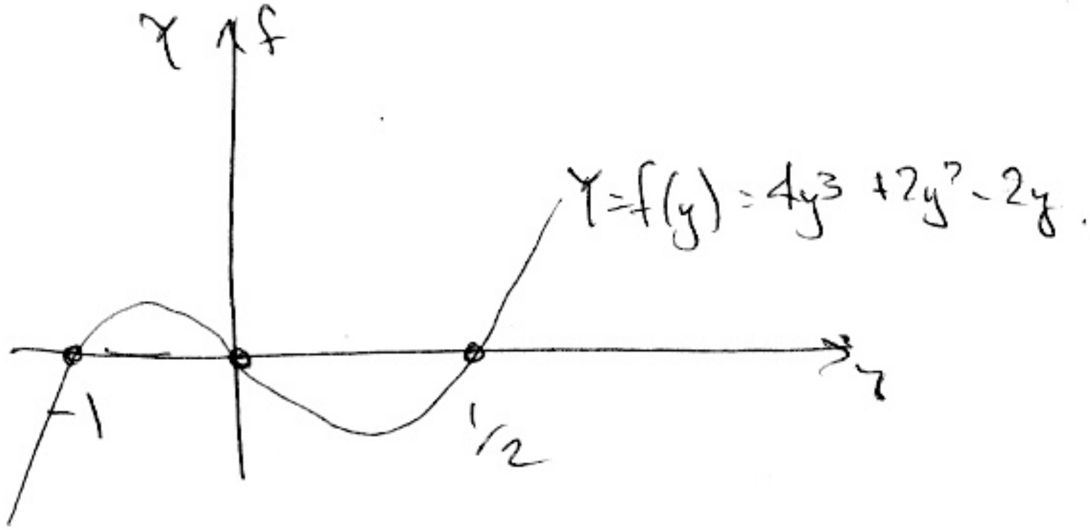
(2) We should find the equilibrium sol'ns: $\frac{dy}{dt} = 0$

(a) Then $4y^3 + 2y^2 - 2y = 0$

i.e. $2(2y^2 + y - 1)y = 0$

i.e. $2(2y-1)(y+1)y = 0$ $\Rightarrow$ $\boxed{y_1 = 0,\ y_2 = \frac{1}{2},\ y_3 = -1}$ are the equilibrium sol'ns.

(b). If we plot: $f(y) = 4y^3 + 2y^2 - 2y$.



(b) $f(y)>0$ on $(-1,0)\cup(\frac{1}{2},\infty)$.

Then $\frac{dy}{dt}>0$, i.e., $y(t)$ is increasing in $(-1,0)\cup(\frac{1}{2},\infty)$

(c) $f(y)<0$ on $(-\infty,-1)\cup(0,\frac{1}{2})$

Then, $\frac{dy}{dt}<0$, i.e., $y(t)$ is decreasing in $(-\infty,-1)\cup(0,\frac{1}{2})$

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Quiz #2. Nombre: ANSWER KEY.

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(1) La función $y(t) = \sin(2t)$, ¿es solución de la Ec. Diferencial?

$$\ddot{y} - \dot{y} + 2\sin(2t) + 2y = 0$$

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(2) Para la ecuación diferencial, encuentre los valores de $y(t)$ para los cuales (a) $y(t)$ está en equilibrio, (b) $y(t)$ es creciente y (c) $y(t)$ es decreciente?.

$$\frac{dy}{dt} = y^3 - y^2 - 12y.$$

ANSWER KEY

(1) We have to compute the derivatives and check is the equality holds: $y = \sin 2t$, $\dot{y} = 2\cos 2t$, $\ddot{y} = -4\sin 2t$.

$$\ddot{y} - \dot{y} + 2\sin(2t) + 2y = -4\sin 2t - 2\cos(2t) + 2\sin 2t + 2\sin 2t$$

$$= -4\sin 2t - 2\cos 2t + 4\sin 2t = -2\cos 2t \neq 0.$$

Then $y(t) = \sin(2t)$ is <u>not</u> a solution.

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(2) (a) We have to find the equilibrium solutions $\frac{dy}{dt} = 0$

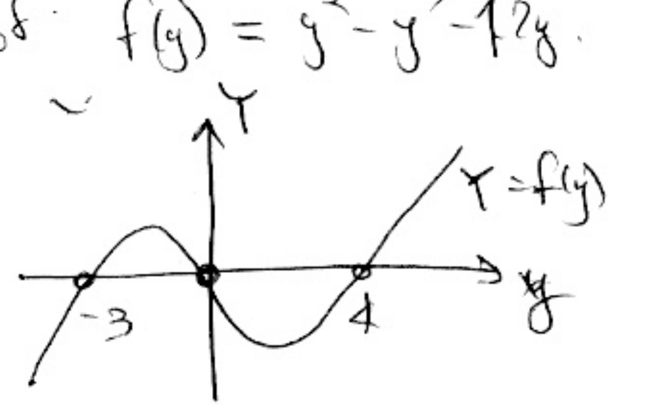
i.e. $y^3 - y^2 - 12y = 0$

i.e. $y(y^2 - y - 12) = 0$

i.e. $y(y-4)(y+3) = 0$

$\Rightarrow$ $\boxed{\begin{array}{l} y_1(t) = 0 \\ y_2(t) = -3 \\ y_3(t) = 4 \end{array}}$ are the equilibrium solns.

(b) We need to plot the graph of $f(y) = y^3 - y^2 - 12y$.



(b) $f(y) > 0$ in $(-3, 0) \cup (4, \infty)$

Then:

$\frac{dy}{dt} > 0$, ie, $y(t)$ is increasing

for $y \in (-3, 0) \cup (4, \infty)$

(c) $f(y) < 0$ in $(-\infty, -3) \cup (0, 4)$.

Then:

$\frac{dy}{dt} < 0$, i.e., $y(t)$ is decreasing

for $y \in (-\infty, -3) \cup (0, 4)$
