UAM - Azcapotzalco | Ecuaciones Diferenciales Ordinarias

Quiz # 3 Nombre: Answer Key. V - 27/Sept/2019.

① Encuentre $g(x)$ para construir una ecuación diferencial de la forma:

$$\frac{d^2x}{dt^2} = -4x + 4t^2 + g(x),$$ ($x(t)$, x es función de t.)

si sabemos que $x(t) = \cos(2t)$ es solución

② Resuelva el problema de valores iniciales: ($x(t)$, x es función de t)

$$\frac{dx}{dt} = \frac{8t^2}{x + 8t^3 x}, \quad x(0) = -2.$$

③ Encuentre la solución de la ecuación diferencial:

$$\frac{dx}{dt} = 2 - 4x - t + 2tx.$$

(Aquí, $x(t)$, x es función de t)

① Como $x(t) = \cos(2t)$ es solución; entonces $t = \frac{\mathrm{Arccos}(x)}{2}$

$$-4\cos(2t) = -4\cos(2t) + 4t^2 + g(x).$$

$$\Rightarrow \quad g(x) = -4t^2 = -4\left(\frac{\mathrm{Arccos}^2(x)}{4}\right) = -\mathrm{Arccos}^2(x)$$

$$\Rightarrow \quad \boxed{\frac{d^2x}{dt} = -4x + 4t^2 - \mathrm{Arccos}^2(x)}$$

② Es una Ec. Dif. separable, $\frac{dx}{dt} = \frac{8t^2}{x(1+8t^3)}$

$$\Rightarrow \int x\,dx = \int\left(\frac{8t^2}{1+8t^3}\right)dt =$$

$$\Rightarrow \frac{1}{2}x^2 = \frac{1}{3}\int \frac{24t^2}{1+8t^3}\,dt = \frac{1}{3}\log|1+8t^3| + C_1$$

$$\Rightarrow \quad x^2 = \frac{2}{3}\log|1+8t^3|^{2/3} + C_2$$

$$\int = \int$$

$x(0) = -2. \Rightarrow \quad (-2)^2 = \frac{2}{3}\text{Log}|1+0|^{2/3} + C_2$

$\Rightarrow \quad \boxed{C_2 = 4}$ and $\boxed{x(t) = -\sqrt{\frac{2}{3}\text{Log}|1+8t^3| + 4}}$

where we choose the "$-$" sign, since $x(0) < 0$

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(3) It is also separable:

$$\frac{dx}{dt} = 2(1-2x) - t(1-2x) = (2-t)(1-2x)$$

$$\Rightarrow \int \frac{1}{1-2x}\,dx = \int (2-t)\,dt. \Rightarrow -\frac{1}{2}\text{Log}|1-2x| = 2t - \frac{t^2}{2} + C_1$$

$$\Rightarrow \text{Log}|1-2x| = -4t + t^2 + C_2$$

$$\Rightarrow \quad 1-2x = C_3 e^{t^2-4t}.$$

$$\Rightarrow \quad \boxed{x(t) = C_4\, e^{t^2-4t} + \frac{1}{2}}$$

UAM- AZCAPOTZALCO. ECUACIONES DIFERENCIALES ORDINARIAS

Quiz #3. Nombre: ANSWER KEY V- 27/Sept/19

(1) Encuentre $h(y)$ para construir una ecuación diferencial de la forma:

$$\frac{d^2y}{dt^2} = 4y + 4t^2 + h(y),$$

si sabemos que $y(t) = e^{2t}$ es solución

(2) Resuelva el problema de valores iniciales:

$$\frac{dy}{dt} = 4ty^2 + 6t^2y^2, \quad y(1) = -\frac{1}{2}$$

(3) Encuentre la solución de la ecuación diferencial.

$$\frac{dy}{dt} = 2 - 4y - t + 2ty$$

SOLUTION KEY

(1) Substitute $y(t) = e^{2t}$ into the equation

$$4e^{2t} = 4e^{2t} + 4t^2 + h(y). \Rightarrow 4t^2 + h(y) = 0$$

$$\Rightarrow h(y) = -4t^2 = -4\left(\frac{1}{2}\operatorname{Log} y\right)^2, \text{ since } y = e^{2t}$$

$$\Rightarrow \frac{1}{2}\operatorname{Log}(y) = t$$

$$= -\operatorname{Log}^2 y$$

$$\Rightarrow \boxed{\frac{d^2y}{dt^2} = 4y + 4t^2 - \operatorname{Log}^2 y}$$

(2) It is a separable equation: $\frac{dy}{dt} = y^2(4t + 6t^2)$

$$\Rightarrow \int \frac{1}{y^2}\,dy = \int (4t + 6t^2)\,dt \Rightarrow -\frac{1}{y} = 2t^2 + 2t^3 + C$$

If $t = 1,\ y = -\frac{1}{2} \Rightarrow 2 = 2 + 2 + C \Rightarrow \boxed{C = -2}$

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$$-\frac{1}{y} = 2t^2 + 2t^3 - 2$$

$$\boxed{y(t) = \frac{1}{2(1-t^2-t^3)}}$$

③ It is also a separable equation.

$$\frac{dy}{dt} = 2(1-2y) - t(1-2y) = (2-t)(1-2y)$$

$$\int \frac{1}{1-2y}\,dy = \int 2 - t\,dt = 2t - \frac{t^2}{2} + C.$$

$$\Rightarrow \quad -\frac{1}{2}\operatorname{Log}|1-2y| = 2t - \frac{t^2}{2} + C_1$$

$$\operatorname{Log}|1-2y| = -4t + t^2 + C_2$$

$$1-2y = C_3\, e^{t^2-4t}.$$

$$\boxed{y(t) = C_4\, e^{t^2-4t} - \frac{1}{2}}$$