UAM - Azcapotzalco | Ecuaciones Diferenciales | Miércoles 9 de octubre, 201

Quiz #5 : Nombre: Answer Key.

(1) Resuelva la ecuación diferencial.

$$2xy^3 + y^4 + (xy^3 - 2)\frac{dy}{dx} = 0$$

(1) We have: $M(x,y) + N(x,y)\frac{dy}{dx} = 0$.

with $M(x,y) = (2xy^3 + y^4)$ and $N(x,y) = (xy^3 - 2)$

Computing the partial derivatives.

$$\left.\begin{aligned}\frac{\partial M}{\partial y} &= \frac{\partial}{\partial y}(2xy^3 + y^4) = 6xy^2 + 4y^3\\ \frac{\partial N}{\partial x} &= \frac{\partial}{\partial x}(xy^3 - 2) = y^3\end{aligned}\right\} \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

We need and integrating factor $\mu = \mu(x,y)$ which should satisfy:

either a) $\frac{1}{\mu}\frac{d\mu}{dx} = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{6xy^2 + 4y^3 - y^3}{xy^3 - 2} = \underbrace{\frac{6xy^2 + 3y^3}{xy^3 - 2}}\ldots$

Function of both x and y: it does not work

or (b) $\frac{1}{\mu}\frac{d\mu}{dy} = \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M+y} = \frac{y^3 - (6xy^2 + 4y^3)}{2xy^3 + y^4}$

$$= \frac{-6xy^2 - 3y^3}{2xy^3 + y^4} = \frac{-3y^2(2x+y)}{y^3(2x+y)}$$

$= -\frac{3}{y}$. Function of y only. It works!

$$\Rightarrow \boxed{\frac{1}{\mu}\frac{d\mu}{dy} = -\frac{3}{y}} \Rightarrow \boxed{\mu(y) = y^{-3}}$$

$= 1 =$

Multiply by $\mu(y) = y^{-3}$ our Diff. Eq:

$$\underbrace{(2x + y)}_{M_{new}(x,y)} + \underbrace{\left(x - \frac{2}{y^3}\right)}_{N_{new}(x,y)} \frac{dy}{dx} = 0$$

$$M_{new}(x,y) + N_{new}(x,y) \frac{dy}{dx} = 0$$

$\left.\begin{array}{l} \dfrac{\partial M_{new}}{\partial y} = 1 \\ \dfrac{\partial N_{new}}{\partial x} = 1 \end{array}\right\}$ Then, it is exact Diff. Eq'n!

Hence, there is an $F(x,y)$, such that:

$\dfrac{\partial F}{\partial x} = \overbrace{2x + y}^{M(x,y)} \Rightarrow \underbrace{F(x,y) = x^2 + xy + g(y)}$

$\Downarrow$

$\dfrac{\partial F}{\partial y} = \overbrace{x - \dfrac{2}{y^3}}^{N(x,y)}$ | $\dfrac{\partial F}{\partial y} = x + g'(y)$

Comparing the two $\frac{\partial F}{\partial y}$: $-\dfrac{2}{y^3} = g'(y)$

$\Rightarrow g'(y) = -2y^{-3}$

$\Rightarrow g(y) = y^{-2} \Rightarrow F(x,y) = x^2 + xy + \dfrac{1}{y^2}$

and the implicit solution is:

$$\boxed{x^2 + xy + \frac{1}{y^2} = C}$$

for a constant C.

UAM - AZCAPOTZALCO $\boxed{\text{ECUACIONES DIFERENCIALES}}$ Miércoles, 9 de octubre, 2019.

Quiz #5 : Nombre: ANSWER KEY

① Resuelva la ecuación diferencial:

$$2y^3 + 2 + 3xy^2 \frac{dy}{dx} = 0$$

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① We have: $M(x,y) + N(x,y)\frac{dy}{dx} = 0$

with $M(x,y) = (2y^3+2)$ and $N(x,y) = 3xy^2$

Computing the partial derivatives:

$$\left.\begin{aligned} \frac{\partial M}{\partial y} &= \frac{\partial}{\partial y}(2y^3+2) = 6y^2 \\ \frac{\partial N}{\partial x} &= \frac{\partial}{\partial x}(3xy^2) = 3y^2 \end{aligned}\right\} \quad \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

We need an integrating factor $\mu(x,y)$.

The integrating factor should satisfy:

(a) $$\frac{1}{\mu}\frac{d\mu}{dx} = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{6y^2 - 3y^2}{3xy^2} = \frac{3y^2}{3xy^2} = \frac{1}{x}$$

which is function of x, only. It works!

$$\Rightarrow \boxed{\frac{1}{\mu}\frac{d\mu}{dx} = \frac{1}{x}} \Rightarrow \boxed{\mu(x) = x}$$

The new Diff Eq'n Becomes:

$$\underbrace{(2xy^3 + 2x)}_{M_{new}(x,y)} + \underbrace{(3x^2y^2)}_{N_{new}(x,y)} \frac{dy}{dx} = 0$$

$$M_{new}(x,y) + N_{new}(x,y)\frac{dy}{dx} = 0$$

$\frac{\partial M_{new}}{\partial y} = (2xy^3 + 2x) = 6xy^2$

$\frac{\partial N_{new}}{\partial x} = \frac{\partial}{\partial x}(3x^2y^2) = 6xy^2$

They are the same! It is an exact Diff. Eq'n!

There is an $F(x,y)$, such that:

$\frac{\partial F}{\partial x} = M_{new}(x,y) = 2xy^3 + 2x \Rightarrow F(x,y) = x^2y^3 + x^2 + f(y)$

$\frac{\partial F}{\partial y} = N_{new}(x,y) = 3x^2y^2$ | $\frac{\partial F}{\partial y} = 3x^2y^2 + f'(y)$

Comparing the $\frac{\partial F}{\partial y}$: $3x^2y^2 = 3x^2y^2 + f'(y)$

$\Rightarrow 0 = f'(y) \Rightarrow f(y) = C_1$ - const.

Then: $F(x,y) = x^2y^3 + x^2 + C_1$

Then, the solution is $F(x,y) = C_2$ i.e.

$x^2y^3 + x^2 + C_1 = C_2$

$\Rightarrow \boxed{x^2y^3 + x^2 = C}$