UAM-AZCAPOTZALCO, EDOs II, Viernes, Octubre 18, 2019

Quiz #6 Nombre: ANSWER KEY.

(1) Se tiene en un contenedor, una muestra de 0.1 gr de una bacteria que se alimenta del caldo en el contenedor. Después de 2 hrs, se tiene 0.15 gr. ¿Cuánto tiempo necesita la bacteria para duplicarse?

(2). Le sirven una limonada fría en un día caluroso de 30°C. 5 minutos después, su limonada está a 10°C, y después de otros 5 minutos está a 15°C. ¿A qué temperatura le sirvieron su limonada?

(1) We use the Malthus growing law: $\frac{dM}{dt} = \alpha M$.

We have the initial population $M(0) = \frac{10}{100} gr = \frac{1}{10} gr$.

and the mass of the bacteria at time $= t$ is.

$$M(t) = M(0) e^{\alpha t}.$$

We use the other condition to determine α: $M(2) = \frac{15}{100}$

Hence: $M(0) e^{2\alpha} = M(2)$, i.e. $\frac{10}{100} e^{2\alpha} = \frac{15}{100}$

$e^{2\alpha} = \frac{15}{10}$, $\boxed{\alpha = \frac{1}{2} \log \frac{15}{10}} \approx 0.2027 \; 1/hr$

Hence: $M(t) = \frac{1}{10} e^{\left[\frac{1}{2}\log\left(\frac{15}{10}\right)\right] t} = \frac{1}{10} e^{\log\left[\left(\frac{15}{10}\right)^{t/2}\right]} = \frac{1}{10}\left(\frac{15}{10}\right)^{t/2}$.

$\boxed{M(t) = \frac{1}{10}\left(\frac{15}{10}\right)^{t/2}}$ We need T hrs for $M(T) = 2M(0)$.

$= 1$:

$$\frac{1}{10}\left(\frac{15}{10}\right)^{T/2} = 2\cdot\frac{1}{10} \Rightarrow \left(\frac{15}{10}\right)^{T/2} = 2.$$

$$\Rightarrow \frac{T}{2}\cdot \mathrm{Log}\left(\frac{15}{10}\right) = \log 2 \Rightarrow \boxed{T = \frac{2\cdot \mathrm{Log}\,2}{\mathrm{Log}\left(\frac{15}{10}\right)}\ \text{hrs}}$$

$$\boxed{T \approx 3.419\ \text{hrs} \approx 3\,h : 25' : 08.48''}$$

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(2) We have to solve Newton's cooling law:

$$\frac{dT}{dt} = -k\,(T - T_a), \text{ where } \begin{array}{l} T(0) = 10^\circ C \\ T(5) = 15^\circ C. \\ \text{and } T_a = 30^\circ C \end{array}$$

The solution is

$$T(t) = (T(0) - T_a)e^{-kt} + T_a, \quad (*)$$

We have to find, k, cooling constant, and $T(-5)$, the temperature at $t = -5$ minutes, from the first measure of temperature. Eq

Eq'n $(*)$ becomes $T(t) = -20e^{-kt} + 30, {}^\circ C$

At $t = 5$ min: $-20e^{-5k} + 30 = 15^\circ$

$$\Rightarrow e^{-5k} = \frac{-15}{-20} = \frac{3}{4} \Rightarrow e^{5k} = \frac{4}{3}$$

$$\Rightarrow \boxed{k = \frac{1}{5}\mathrm{Log}\left(\frac{4}{3}\right)\ 1/\text{min}} \approx 0.058\ 1/\text{min}$$

$$T(t) = -20e^{-\left(\frac{1}{5}\log\left(\frac{4}{3}\right)\right)t} + 30 = -20\left(\frac{4}{3}\right)^{-t/5} + 30, {}^\circ C.$$

$$\boxed{T(t) = -20\left(\frac{4}{3}\right)^{-t/5} + 30, {}^\circ C}$$

Now,

$$T(-5) = -20\left(\frac{4}{3}\right)^{-(-5)/5} + 30 = -20\left(\frac{4}{3}\right)^{1} + 30$$

$$= -\frac{80}{3} + 30 = \frac{90-80}{3} = \frac{10}{3}$$

$$\Rightarrow \boxed{T(-5) = \frac{10}{3}\,^\circ C \approx 3.33\,^\circ C}$$