UAM-AZCAPOTZALCO — ECUACIONES DIFERENCIALES — Viernes, 8 Nov 2019

Quiz #8 - #9 Nombre

**Quiz #8.** Resuelva el problema de valores iniciales.

$$\ddot{y} - 6\dot{y} + 10y = 0, \quad y(0) = 2, \quad \dot{y}(0) = -2$$

**Quiz #9.** Escriba la forma de la solución particular de la Ec. Diferencial.

$$\ddot{y} - 4\dot{y} + 4y = t^4 e^{-2t}$$

**Quiz #8** This is a 1) Linear, 2) Constant Coefficients and 3) homogeneous Diff Eq: look for solutions $y(t) = e^{rt}$.

$$r^2 - 6r + 10 = 0.$$

$$\Rightarrow (r-3)^2 + 1 = 0 \quad \Rightarrow \quad r_{1,2} = 3 \pm i$$

Complex Solution $y(t) = e^{rt} = e^{(3+i)t} = e^{3t} e^{it} = e^{3t}(\cos t + i \sin t)$

Then, the solution is $\boxed{y(t) = (C_1 \cos t + C_2 \sin t) e^{3t}}$

It remains to find $C_1$ and $C_2$ using the Initial Conditions:

Compute: $\dot{y}(t) = (-C_1 \sin t + C_2 \cos t) e^{3t} + 3(C_1 \cos t + C_2 \sin t) e^{3t}$

$2 = y(0) = C_1 \quad \Rightarrow C_1 = 2$

$-2 = \dot{y}(0) = C_2 + 3C_1 \quad \Rightarrow C_2 = -2 - 3C_1 = -8 \quad \boxed{C_2 = -8}$

$$\boxed{y(t) = (2\cos t - 8 \sin t) e^{3t}}$$

The solution is:

$$\boxed{y(t) = \left(2\cos(t) - 8\sin(t)\right) e^{3t}}$$

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Quiz # 9.) Solve $\ddot{y} - 4\dot{y} + 4y = t^4 e^{-2t}$

Step ① 1st Solve: $\ddot{y}_h - 4\dot{y}_h + 4y_h = 0$

This is a 1) Linear, 2) Constant Coef's Diff Eq, 3) Homogeneous

$\Rightarrow y(t) = e^{rt} \Rightarrow r^2 - 4r + 4 = 0 \Rightarrow (r-2)^2 = 0$

$r_{1,2} = 2 \Rightarrow y_h(t) = e^{2t}(C_1 + t C_2)$

Step ② Propose

$$y_p(t) = \left(At^4 + Bt^3 + Ct^2 + Dt + E\right) e^{-2t}$$

we do not repeat solutions in $y_h(t)$. Then this is the particular solution.

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