

## Quiz # 6 | : Nombre:

ANSWER KEY.

Instrucciones: Para recibir puntos:

- 1) Responda correctamente
- 2) Escriba en forma clara y concisa
- 3) Entregue su trabajo limpio y con sus ideas en orden.
- 4) Muestre todas sus ventajas. Simple & que.
- 5) Explique, argumente y justifique sus respuestas
- 6) Problemas sin desarrollo, explicación, argumento o justificación, vale  cero  puntos.

① Resuelva la ecuación diferencial:

$$\frac{dy}{dx} = -\frac{(3x^2 + y)}{x^2y - x}$$

SOLUTION KEY

① Let us write the Diff Eq as:

$$(3x^2 + y) + (x^2y - x) \frac{dy}{dx} = 0$$

Then  $M(x,y) = 3x^2 + y$  &  $N(x,y) = x^2y - x$ .Hence:  $\frac{\partial M}{\partial y} = 1 \neq \frac{\partial N}{\partial x} = 2xy - 1$ . It is not exact.(2) If  $\mu = \mu(x)$  only:

$$\begin{aligned} \frac{1}{\mu} \frac{d\mu}{dx} &= \frac{M_y - N_x}{N} = \frac{1 - (2xy - 1)}{x^2y - x} = \frac{2 - 2xy}{x^2y - x} \\ &= \frac{2(1 - xy)}{x(xy - 1)} = \frac{-2(xy - 1)}{x(xy - 1)} = -\frac{2}{x} \end{aligned}$$

= 1 =

Then:  $\frac{1}{x} \frac{du}{dx} = \frac{-2}{x} \Rightarrow \frac{d}{dx} \log|u| = -2 \frac{d}{dx} (\log|x|)$

$\Rightarrow \log|u| = \log|u|^{-2} \Rightarrow \boxed{\mu(x) = x^{-2}}$

Multiply the Diff Eqn by  $\mu(x) = x^{-2}$ .

$x^{-2}(3x^2 + y) + x^{-2}(x^2y - x) \frac{dy}{dx} = 0$  is now exact.

$\boxed{\left(3 + \frac{y}{x^2}\right) + \left(y - \frac{1}{x}\right) \frac{dy}{dx} = 0}$

It is exact!

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y} \left(3 + \frac{y}{x^2}\right) = \frac{1}{x^2}$

$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x} \left(y - \frac{1}{x}\right) = \frac{1}{x^2}$  Exact!

Then, find  $F(x,y)$  such that.

(\*)  $\frac{\partial F}{\partial x} = 3 + \frac{y}{x^2}$

$\frac{\partial F}{\partial y} = \left(y - \frac{1}{x}\right) \Rightarrow F(x,y) = \frac{1}{2}y^2 - \frac{y}{x} + h(x)$

$\Rightarrow \frac{\partial F}{\partial x} = 0 + \frac{y}{x^2} + h'(x)$

Compare with (\*) above:

$\Rightarrow h'(x) = 3 \quad \boxed{h(x) = 3x}$

$\Rightarrow F(x,y) = \frac{1}{2}y^2 - \frac{y}{x} + 3x$

Solution:  $F(x,y) = C$  because

$\boxed{\frac{1}{2}y^2 - \frac{y}{x} + 3x = C}$