UAM - AZCAPOTZALCO | ECUACIONES DIFERENCIALES ORDINARIAS | Viernes 14/Feb/2020

Quiz #7: Nombre: ANSWER KEY.

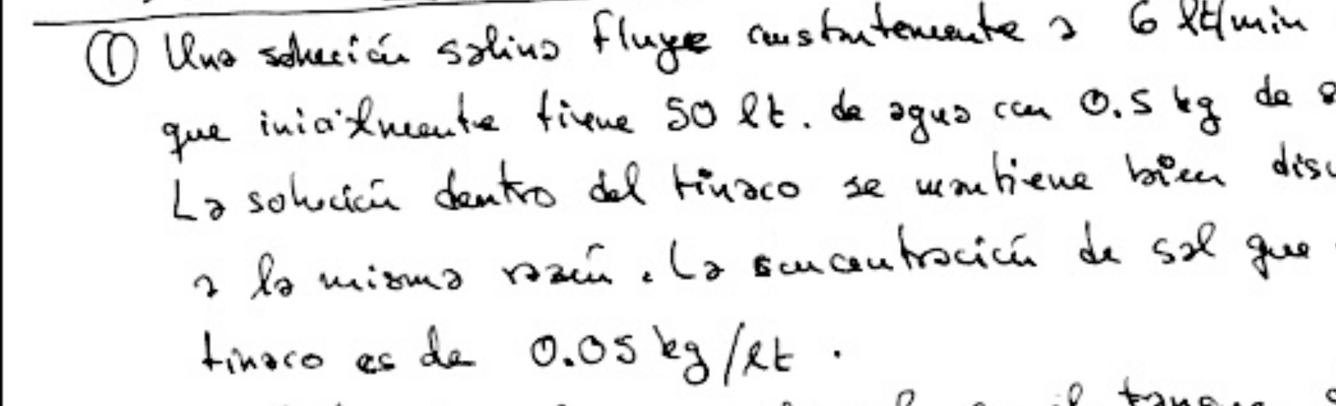
Instrucciones: Para recibir puntos:

1) Responda correctamente.
2) Escriba de forma clara y concisa.
3) Entregue su trabajo limpio y con sus ideas en orden
4) Muestre todas sus cuentas y simplifique
5) Explique, argumente y justifique sus respuestas
6) Problema sin desarrollo, explicación, argumento o justificación, vale 0 puntos (cero)

① Una solución salina fluye constantemente a 6 lt/min a un tinaco que inicialmente tiene 50 lt. de agua con 0.5 kg de sal disuelta. La solución dentro del tinaco se mantiene bien disuelta y sale a la misma razón. La concentración de sal que entra al tinaco es de 0.05 kg/lt.

Determine la masa de sal en el tanque después de t minutos. ¿Cuándo la concentración de sal será 0.03 kg/l?

$\boxed{\text{SOLUTION KEY}}$



$V(t)$: Volume inside the container

$C(t)$ - concentration at time t. $= \dfrac{\text{Mass}(t)}{\text{Volume}(t)}$

F_{in} - ~~Flow~~ Input Flux

F_{out} - output Flux:

$M(t)$ = Mass at time t inside the tank.

Then, the equation for the mass $M(t)$ is

$$\frac{dM}{dt} = F_{in} C_{in} - F_{out} C_{out}$$

$$= \frac{6\, lt}{min} \cdot \frac{5}{100} \frac{kg}{lt} - \frac{6\, lt}{min} \cdot \frac{M(t)\, kg}{V(t)\, lt}$$

$$= \frac{30}{100} \frac{kg}{min} - \frac{6}{V(t)} M(t) \frac{kg}{min}.$$

Now, since $F_{in} = F_{out} \Rightarrow V(t) = \text{Const} = V(0) = 50\, lt.$

Hence, we have to solve the Diff Eqn.

$$\frac{dM}{dt} = \frac{30}{100} - \frac{6}{50} M.$$

i.e. $$\frac{dM}{dt} + \frac{12}{100} M = \frac{30}{100}$$

Integrating factor: $$\mu = e^{\int \frac{12}{100} dt} = e^{\frac{12}{100} t}.$$

$$\int \mu(t) f(t)\, dt = \int e^{\frac{12}{100} t} \cdot \frac{30}{100} dt \qquad \frac{100}{12} \cdot \frac{30}{100} e^{\frac{12}{100} t}$$

$$= \frac{30}{12} e^{\frac{12}{100} t}$$

Then, the solution $y(t) = \frac{1}{\mu(t)} \int \mu(t) f(t)\, dt + \frac{c}{\mu(t)}$

Becomes $$M(t) = \frac{1}{e^{\frac{12}{100} t}} \frac{30}{12} e^{\frac{12}{100} t} + C e^{-\frac{12}{100} t}$$

$$\Rightarrow \boxed{M(t) = C e^{-\frac{12}{100} t} + \frac{30}{12}\ kg}$$

At $t=0$, $M(0) = \frac{1}{2}$ kg.

Then $\frac{1}{2} = M(0) = C + \frac{30}{12} \Rightarrow C = \frac{1}{2} - \frac{30}{12}$

$C = -\frac{24}{12} = -2$.

$$\Rightarrow \boxed{M(t) = -2e^{-\frac{12}{100}t} + \frac{30}{12} \text{ kg}}$$

Now, the concentration is

$$C(t) = \frac{M(t)}{Vol} = \frac{M(t)}{50\,lt} = \left(-\frac{2}{50}e^{-\frac{12}{100}t} + \frac{30}{50\cdot 12}\right)\frac{kg}{lt}$$

$$\boxed{C(t) = \left(-\frac{4}{100}e^{-\frac{12}{100}t} + \frac{5}{100}\right)\frac{kg}{lt}}$$

We know: $C(t) = 0.03 \frac{kg}{lt} = \frac{3}{100} \frac{kg}{lt}$.

We need to know t:

$$-\frac{4}{100}e^{-\frac{12}{100}t} + \frac{5}{100} = \frac{3}{100}$$

$$-4e^{-\frac{12}{100}t} + 5 = 3. \Rightarrow 5-3 = 4e^{-\frac{12}{100}t}.$$

$$2 = 4e^{-\frac{12}{100}t} \Rightarrow e^{+\frac{12}{100}t} = 2 \Rightarrow +\frac{12}{100}t = \ln 2.$$

$$\Rightarrow \boxed{t = \frac{100}{12}\ln 2 \text{ min.}}$$

Or: $\boxed{t \approx 5.776 \text{ min}}$ $\approx 5':46''$.