UAM-AZCAPOTZALCO | Ecuaciones Diferenciales Ordinarias | Viernes, Feb. 21, 2020

Quiz #8: Nombre: ANSWER KEY.

Instrucciones: Para recibir puntos:

1) Responda correctamente.
2) Escriba de forma clara y concisa.
3) Entregue su trabajo limpio y con sus ideas en orden
4) Muestre todos sus cuentas y simplifique
5) Explique, argumente y justifique sus respuestas (cero)
6) Problemas sin desarrollo, explicación, argumento o justificación, vale 0 puntos

(1) Resuelva el problema de valores iniciales:

$$\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 16y = 0 \quad \Big| \quad \begin{matrix} y(0) = 0 \\ \dot{y}(0) = 7 \end{matrix}$$

(2) Resuelva la Ec. Diferencial:

$$t^2\frac{dy}{dt} + \frac{2}{3}ty = \frac{4}{3}t^3 y^{-2}.$$

SOLUTION KEY.

(1) The Diff Eq: $\ddot{y} + 8\dot{y} + 16y = 0$ is
1) Linear
2) Constant Coeff's
3) Homogeneous

Then, look for solutions of the form:
$y(t) = e^{rt}$, $r = \text{const}$: $r^2 + 8r + 16 = 0$

i.e. $(r+4)^2 = 0 \Rightarrow r_1 = r_2 = -4$.

We have repeated roots. The solutions are then:

$$y_1(t) = e^{-4t}, \quad y_2(t) = te^{-4t}.$$

and the general solution is:

$$y(t) = C_1 e^{-4t} + C_2 te^{-4t}.$$

or $$\boxed{y(t) = (C_1 + C_2 t)e^{-4t}}$$

Using the initial conditions:

$$y(0) = (C_1 + 0)\cdot 1 = 0 \Rightarrow \boxed{C_1 = 0}$$

$$\Rightarrow y(t) = C_2 t e^{-t}.$$

$$\dot{y}(t) = C_2 \dot{e}^{-t} + C_2 t(-1)e^{-t} = C_2(1-t)e^{-t}$$

Using again the initial condit:

$$\dot{y}(0) = C_2(1-0)\cdot 1 = 7 \Rightarrow \boxed{C_2 = 7}$$

$$\boxed{y(t) = 7te^{-t}}$$

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② The Diff Eq'n

$$t^2\frac{dy}{dt} + \frac{2}{3}ty = \frac{4}{3}t^3y^{-2}$$

is a Bernoulli eq'n. Let $v(t) = y^\alpha(t)$. Then:

$$\frac{dv}{dt} = \alpha y^{\alpha-1}\frac{dy}{dt} = \alpha t \Rightarrow \alpha y^{\alpha-1}t^2\frac{dy}{dt} + \alpha y^{\alpha-1}\frac{2t}{3}y = \alpha y^{\alpha-1}\frac{4}{3}t^3y^{-2}$$

$$\Rightarrow t^2\underbrace{\alpha y^{\alpha-1}\frac{dy}{dt}}_{\frac{dv}{dt}} + \alpha\frac{2}{3}t\underbrace{y^\alpha}_{v} = \alpha\frac{4}{3}t^3y^{\alpha-3}.$$

Choose $\alpha - 3 = 0$. Then, $\alpha = 3$ and:

$$t^2\frac{dv}{dt} + 3\cdot\frac{2}{3}tv = 3\cdot\frac{4}{3}t^3y^0.$$

$$\Rightarrow t^2\frac{dv}{dt} + 2tv = 4t^3 \Rightarrow \frac{d}{dt}(t^2v) = 4t^3.$$

$$\Rightarrow t^2v(t) = t^4 + C \Rightarrow v(t) = t^2 + \frac{C}{t^2}$$

$$\Rightarrow v(t) = y^3(t) \Rightarrow \boxed{y(t) = \sqrt[3]{v(t)} = \sqrt[3]{t^2 + \frac{C}{t^2}}}$$