ANSWER KEY.

EVALUACIÓN GLOBAL DE ECUACIONES DIFERENCIALES ORDINARIAS. TURNO VESPERTINO

Trimestre 19-O Marzo 17 de 2020. Horario: 16:00-17:30 h Grupo: ....

Alumno: ....KEY.............. **Matrícula:** ....

El examen global consta de los problemas marcados con (♣). Quien presente una de las partes, deberá resolver todos los problemas correspondientes a esa parte. Los resultados deberán mostrar el procedimiento respectivo.

PRIMERA PARTE

1. ♣(10 puntos) Resolver la siguiente ecuación diferencial

$$y' + \frac{y}{x+1} - \cos x = 0$$

2. ♣(10 puntos) Resolver la siguiente ecuación diferencial

$$(2xy + 2y^2 + 4y)\,dx + (y^2 - x^2 - 4x - 1)\,dy = 0, \quad y(0) = 1$$

3. ♣(10 puntos) Resolver la siguiente ecuación diferencial

$$x^2y' + 2y - y^{\frac{1}{2}} = 0$$

4. ♣(10 puntos) Un tanque cuya capacidad es de 300 galones contiene 200 galones de líquido en el cual se disuelven 10 libras de sal. Una salmuera que contiene 2 libras de sal por galón se bombea al tanque con una rapidez de 7 galones por minuto. La solución mezclada se bombea hacia afuera del tanque con una rapidez de 3 galones por minuto. a) Plantear el problema con valores iniciales. b) Hallar el número de libras $A(t)$ de sal que hay en el tanque después de 15 minutos. c) Hallar el número de libras $A(t)$ de sal que hay en el tanque en el instante en que está a punto de derramarse el líquido.

SEGUNDA PARTE

5. Resolver el siguiente problema de valores iniciales:

$$2y'' + 8y' + 6y = 0, \quad y(0) = -1, \quad y'(0) = 2$$

6. ♣(10 puntos) Dada la ecuación diferencial siguiente

$$(2x+1)\,y'' + 4x\,y' - 4y = 0$$

y **una de sus soluciones** $y_1(x) = e^{-2x}$, encontrar otra solución linealmente independiente de la solución dada y obtener la solución general.

7. ♣(15 puntos) Resolver la siguiente ecuación diferencial:

$$y'' - y' - 2y = x + 3e^{-x}$$

8. ♣(15 puntos) Resolver la ecuación:

$$y'' + 25y = \tan(5x)\sec(5x)$$

TERCERA PARTE

9. ♣(10 puntos) Un cuerpo que pesa 8 lb alarga 8 ft un resorte, el cual está suspendido del techo. Al inicio el cuerpo se suelta desde un punto que está $\frac{1}{2}$ ft abajo de su posición de equilibrio, con una velocidad dirigida hacia abajo de $\frac{3}{2}$ pies/s

   a) Plantear el problema de valores iniciales y determinar el desplazamiento del cuerpo en función del tiempo.

   b) Expresar la solución en su forma alternativa, $R\cos(\omega_0 t \pm \delta)$ o R sen $(\omega_0 t \pm \delta)$.

   c) Obtener el instante en el que el cuerpo pasa por seguida vez por su posición de equilibrio y determinar la dirección en que se dirige en ese instante.

10. ♣(10 puntos). Una masa de 20 g hace que se estire un resorte una dstancia de 5 cm. El resorte está conectado a un amortiguador de aceite cuya constante de amortiguamiento es de 400 dinas seg/cm. Se tira de la masa hacia abajo una distancia de 2 cm y se suelta.

    a) Plantear el problema de valores iniciales y determinar el desplazamiento de la masa en función del tiempo.

    b) Expresar la solución en su forma alternativa.

UAM-AZCAPOTZALCO | Ecuaciones Diferenciales Ordinarias I | Martes, Marzo 17, 2020

EXAMEN GLOBAL. Trimestre: 19-O. ANSWER KEY

PRIMERA PARTE

(1) Resolver la siguiente ecuación diferencial

$$y' + \frac{y}{x+1} = \cos x.$$

It is a first order, linear ODE. $\mu' = \frac{1}{x+1}\mu \Rightarrow \log \mu = \log|1+x|$

$\Rightarrow \mu(x) = 1+x$. Then,

$$\mu \quad (x+1)y' + y = (x+1)\cos x \Rightarrow \int ((x+1)y)' dx = \int (x+1)\cos x \, dx$$

By parts

$$(x+1)\,y(x) = (x+1)\sin x + \cos x + C \qquad \boxed{y(x) = \sin x + \frac{\cos x}{x+1} + \frac{C}{x+1}}$$

(2) Solve:

$$\underbrace{2xy + 2y^2 + 4y}_{M(x,y)} + \underbrace{(y^2 - x^2 - 4x - 1)}_{N(x,y)}\frac{dy}{dx} = 0$$

$$\left.\begin{aligned} \frac{\partial M}{\partial y} &= 2x + 4y + 4 \\ \frac{\partial N}{\partial x} &= -2x - 4 \end{aligned}\right\}$$ Not identical, look for an Integrating factor

$$(\mu M)_y = (\mu M)_x.$$

If $\mu = \mu(x)$: $\frac{\mu_x}{\mu} = \frac{M_y - N_x}{N} = \frac{4x + 4y + 8}{y^2 - x^2 - 4x - 1}$. Doesn't work

If $\mu = \mu(y)$: $\frac{\mu_y}{\mu} = \frac{N_x - M_y}{M} = \frac{-4(x+y+2)}{2y(x+y+2)} = \frac{-2}{y}$

$$\Rightarrow (\log \mu)_y = -2(\log y)_y \Rightarrow \Rightarrow \mu(y) = y^{-2}$$

Then, the Diff Eq'n becomes $y^{-2}M + y^{-2}N\frac{dy}{dx} = 0$

i.e. $\underbrace{(2x+4)y^{-1}+2}_{M(x,y)} + \underbrace{\left(1-(x^2+4x+1)y^{-2}\right)}_{N(x,y)}\frac{dy}{dx} = 0.$

$$\left.\begin{aligned} \frac{\partial M}{\partial y} &= -(2x+4)y^{-2} \\ \frac{\partial N}{\partial x} &= -2x\,y^{-2} - 4y^{-2} = -(2x+4)y^{-2} \end{aligned}\right\} \text{Identical: Exact Eq'n.}$$

$$\frac{\partial F}{\partial x} = (2x+4)y^{-1} + 2$$

$$\frac{\partial F}{\partial y} = 1 - (x^2+4x+1)y^{-2}$$

Integrating both eq's w.r.t. x and y respectively.

$\Rightarrow F(x,y) = (x^2+4x)y^{-1} + 2x + g(y)$

$F(x,y) = y + (x^2+4x+1)y^{-1} + h(x).$

Comparing: $2x = h(x)$ $\Rightarrow F(x,y) = (x^2+4x)y^{-1} + 2x + y + y^{-1}.$

$y + y^{-1} = g(y)$

Then, the implicit solution $F(x,y) = C$ is:

$$\boxed{y + (x^2+4x+1)y^{-1} + 2x = C}$$

Now, for $y(0) = 1$,

$$1 + (0+0+1)1^{-1} + 0 = C \Rightarrow C = 2$$

$$\boxed{y + (x^2+4y+1)y^{-1} + 2x = 2}$$

(3) This is a Bernoulli Diff Eq'n:

$$x^2 \frac{dy}{dx} + 2y = y^{1/2} \qquad \Rightarrow n = \frac{1}{2}$$

$$\Rightarrow y \quad v(x) = \left(y(x)\right)^{1-n} = \left(y(x)\right)^{1/2}.$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2} y^{-1/2} \frac{dy}{dx} = \frac{1}{2} y^{-1/2}\left(\frac{y^{1/2}}{x^2} - \frac{2y}{x^2}\right) = \frac{1}{2x^2} - \frac{y^{1/2}}{x^2}$$

$$\Rightarrow \boxed{\frac{dv}{dx} + \frac{v}{x^2} = \frac{1}{2x^2}.}$$

Integrating factor: $\mu = e^{\int \frac{1}{x^2} dx} = e^{-\frac{1}{x}}$

$$\int b(x)\,\mu(x)\,dx = \int \frac{1}{2x^2} e^{-1/x} dx \underset{\substack{\uparrow \\ z = -\frac{1}{x}}}{=} \frac{1}{2}\int e^z dz = \frac{e^z}{2} = \frac{e^{-1/x}}{2}$$

Hence:

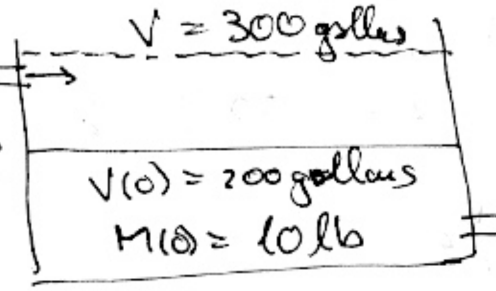
$$v(x) = \frac{1}{\mu(x)} \int \mu(x)\, b(x)\, dx + \frac{c}{\mu(x)} = \frac{1}{e^{-1/x}}\left(\frac{e^{-1/x}}{2}\right) + \frac{c}{e^{-1/x}}$$

$$v(x) = \frac{1}{2} + c\, e^{1/x}$$

$$\Rightarrow \boxed{y(x) = v^2(x) = \left(\frac{1}{2} + c e^{1/x}\right)^2}$$

(4) $v_{in} = 7\,\frac{g}{min}$

$C_{in} = 2\,\frac{lb}{g}$



$v_{out} = 3\,\frac{gallons}{min}$.

$C_{out} = C(t) = \frac{M(t)}{V(t)}$

V - Volume in tank.

M - Mass $\qquad C = \frac{M}{V}$.

C - Concentración.

(a) $\frac{dV}{dt} = (7-3)\,\frac{gallons}{min}$ $\qquad V(t) = 4t + 200. = 4(t+50)$

Diff Eq'n for (t):

$$\frac{dM}{dt} = (\text{Flux-in}) - (\text{Flux out}).$$

$$= v_{in} \cdot C_{in} - v_{out}\, C_{out}.$$

$$= 7 \cdot 2 - 3\,\frac{M(t)}{V(t)}.$$

$$\Rightarrow \boxed{\frac{dM}{dt} + \frac{3\,M(t)}{4(t+50)} = 14 \;;\; M(0) = 10\text{ lb}}$$

(b) $\frac{\mu'}{\mu} = \frac{3}{4(t+50)}$

$(\log \mu)' = \frac{3}{4}(\log(t+50))'$ $\quad \Rightarrow \boxed{\mu(t) = (t+50)^{3/4}}$

$$\Rightarrow (t+50)^{3/4} M(t) = \int (t+50)^{3/4}\, 14\, dt = 8(t+50)^{7/4}$$

$$M(t) = 8(t+50) + C(t+50)^{-3/4}$$

$$M(0) = 400 + \frac{C}{50^{3/4}} = 10 \Rightarrow C = -390(50)^{3/4}$$

$$\boxed{M(t) = 400\left(\frac{t}{50} + 1\right) - \frac{390}{\left(\frac{t}{50} + 1\right)^{3/4}}}$$

Concentration at time t:

$$\boxed{C(t) = \frac{M(t)}{V(t)} = 2 - \frac{39}{20\left(\frac{t}{50}+1\right)^{7/4}}}$$

$$\left(C(t) = \frac{M(t)}{V(t)} = \frac{8(t+50)}{4(t+50)} - \frac{390}{4(t+50)\left(t/50+1\right)^{3/4}} \right.$$

$$\left. = 2 - \frac{390}{4\cdot 50\left(t/50+1\right)\left(t/50+1\right)^{3/4}} = 2 - \frac{390}{200\left(t/50+1\right)^{7/4}} \right)$$

At $t=0$

$$C(0) = 2 - \frac{39}{20} = \frac{40-39}{20} = \frac{1}{20}\ \frac{lb}{gallon}$$ ✓

After 15 min's:

$$M(15) = 400\left(\frac{15}{50}+1\right) - \frac{390}{\left(\frac{15}{50}+1\right)^{3/4}} = 8(15+50) - \frac{390}{\left(\frac{65}{50}\right)^{3/4}}$$

$$\boxed{M(15) = 199.66\ lb \approx 200\ lb}$$

(c) when $V(t) = 300$ $\quad 4(t+50) = 300 \Rightarrow \boxed{t = 25\ min}$

$$M(25) = 400\left(\frac{25}{50}+1\right) - \frac{390}{\left(\frac{25}{50}+1\right)^{3/4}} = 400\left(\frac{75}{50}\right) - \frac{390}{\left(\frac{75}{50}\right)^{3/4}}$$

$$\boxed{M(25) \approx 312.26\ lb}$$

SEGUNDA PARTE

(5) $2y'' + 8y' + 6y = 0.$

$$\Rightarrow \quad y'' + 4y' + 3y = 0 \quad , \quad y(0) = -1 \qquad y'(0) = 2.$$

$$y(x) = e^{rx}: \qquad r^2 + 4r + 3 = 0 \Rightarrow \begin{cases} r_1 = -3 \\ r_2 = -1. \end{cases}$$
$$(r+3)(r+1) = 0$$

$\Rightarrow \quad y(x) = C_1 e^{-3x} + C_2 e^{-x}$ is the general solution.

$$y'(x) = -3C_1 e^{-3x} - C_2 e^{-x}.$$

$$\left.\begin{aligned} y(0) &= C_1 + C_2 = -1 \\ y'(0) &= -3C_1 - C_2 = 2 \end{aligned}\right\} \qquad -2C_1 = 1 \Rightarrow \boxed{C_1 = -\frac{1}{2}}$$

$$\Rightarrow \frac{3}{2} - C_2 = 2 \Rightarrow \boxed{C_2 = -\frac{1}{2}}$$

$$\Rightarrow \quad \boxed{y(x) = -\frac{1}{2}\left(e^{-3x} + e^{-x}\right)}$$

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(6). $(2x+1)y'' + 4xy' - 4y = 0$. $y_1(x) = e^{-2x}$ is sol'n.

$y_1(x) = e^{-2x}$ is solution.

$$(2x+1)(4e^{-2x}) + 4x(-2e^{-2x}) - 4e^{-2x} =$$
$$= ((8x+4) + (-8x) - 4)e^{-2x} = 0$$

Propose:

$y_2(x) = v(x)e^{-2x}$. Find $v(x)$.

$$y_2'(x) = (v' - 2v)e^{-2x}$$

$$y_2''(x) = (v'' - 4v' + 4v)e^{-2x}.$$

Substitute into the Diff. Eq.

$$(2x+1)(v''-4v'+4v)e^{-2x}+4x(v'-2v)e^{-2x}-4ve^{-2x}=0$$

$$(2x+1)v''+(-4(2x+1)+4x)v'+(4(2x+1)-8x-4)=0$$

$$\Rightarrow (2x+1)v''+(-4x-4)v'=0 \quad \Rightarrow \frac{v''}{v'}=\frac{4(x+1)}{2x+1}=2+\frac{2}{2x+1}$$

$$\Rightarrow \quad \operatorname{Log}|v'| = 2x + \log|2x+1|.$$

$$\Rightarrow \quad v'(x) = (2x+1)e^{2x} \Rightarrow$$

$$\Rightarrow \quad v(x) = \int (2x+1)e^{2x}\,dx = (2x+1)\frac{e^{2x}}{2} - 2\frac{e^{2x}}{4} = xe^{2x}$$

$$\Rightarrow \quad y_2(x) = v(x)e^{-2x} = (xe^{2x})e^{-2x} = x.$$

$$\boxed{y_2(x) = x}$$

They are linearly independent, if $x = -\frac{1}{2}$:

$$W[y_1, y_2](x) = \det\begin{pmatrix} e^{-2x} & x \\ -2e^{-2x} & 1 \end{pmatrix} = (1+2x)e^{-2x} \neq 0$$

if $x = -\frac{1}{2}$.

The general solution is.

$$\boxed{y(x) = C_1 e^{-2x} + C_2 x}$$

⑦ $y'' - y' - 2y = x + 3e^{-x}$.

(a) Homogenous eq'n:

$$y_h'' - y_h' - 2y_h = 0$$

$$r^2 - r - 2 = 0 \Rightarrow r_1 = 2, \quad r_2 = -1.$$

$$\boxed{y_h(x) = C_1 e^{2x} + C_2 e^{-x}}$$

(b). Non-homogeneous eq'n: Sub-problem ①.

$$y_p'' - y_p' - 2y_p = x.$$

Propose a particular solution: $y_{p,1}(x) = Ax + B$.

Substitute into the Diff. Eq:

$$0 - (A) - 2(Ax + B) = x.$$

$$-2Ax - (A + 2B) = x.$$

$-2A = 1$
$-(A+2B) = 0$ $\Rightarrow$ $A = -\frac{1}{2}$, $B = \frac{1}{4}$.

$$\Rightarrow y_{p,1}(x) = -\frac{1}{2}\left(x - \frac{1}{2}\right). = -\frac{1}{4}(2x - 1).$$

Subproblem ② $\quad y_p'' - y' - 2y_p = 3e^{-x}$.

If we propose $Y_1(x) = Ae^{-x}$, won't work, since this solves the homogenous equation:

Then $y_{p,2}(x) = Axe^{-x}$

$$y_{p,2}'(x) = A(1-x)e^{-x}$$

$$y_{p,2}''(x) = A(-2+x)e^{-x}$$

Substitute into the Diff Eq'n:

$$A(x-2)e^{-x} - A(1-x)e^{-x} - 2Ax e^{-x} = 3e^{-x}.$$

$$A(x-2-1+x-2x) = 3$$

$$-3A = 3 \Rightarrow \boxed{A=-1}$$

$$y_{p,2} = -xe^{-x}.$$

Then, the solution is.

$$\boxed{y(x) = C_1 e^{2x} + C_2 e^{-x} - \frac{1}{4}(2x-1) - xe^{-x}}$$

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(7) Solve the Diff Eq:

$$y'' + 25y = \underbrace{\tan(5x)\sec(5x)}_{f(x)}.$$

Homogeneous equation

$$y_h'' + 25y_h = 0$$

$$y_h(x) = C_1\cos(5x) + C_2(\sin 5x).$$

$$W = W[y_1,y_2](x) = \det\begin{pmatrix}\cos 5x & \sin 5x \\ -5\sin 5x & 5\cos 5x\end{pmatrix} = 5 \neq 0$$

The solutions $y_1(x) = \cos 5x$, $y_2(x) = \sin(5x)$ are linearly indep

Nonhomogeneous equation.

Particular solution $y_p(x) = A(x)\cos 5x + B(x)\sin 5x$.

$$A(x) = -\int \frac{y_2(x)\, f(x)}{a(x)\, W(x)}\, dx = -\int \frac{\sin(5x)\tan(5x)\sec(5x)}{1 \cdot 5}\, dx.$$

$$= -\frac{1}{5}\int \tan^2(5x)\, dx = -\frac{1}{5}\int \left(\sec^2(5x) - 1\right) dx$$

$$= -\frac{1}{5}\left(\frac{\tan 5x}{5} - x\right) = -\frac{1}{25}\left(\tan 5x - 5x\right)$$

$$B(x) = \int \frac{y_1(x)\, f(x)}{a(x)\, W(x)}\, dx = \frac{1}{5}\int \cos(5x)\tan(5x)\sec(5x)\, dx$$

$$= \frac{1}{5}\int \tan 5x\, dx = -\frac{1}{25}\log|\cos 5x|$$

Non-Homogeneous equation sol'n Particular solution:

$$y_p(x) = -\frac{1}{25}\left(\tan 5x - 5x\right)\cos 5x - \frac{1}{25}\operatorname{Log}|\cos 5x|\sin(5x).$$

$$= \underbrace{-\frac{1}{25}\sin 5x}_{\text{Solves the homogeneous eq'n}} + \frac{x}{5}\cos 5x - \frac{1}{25}\operatorname{Log}|\cos 5x|\sin(5x)$$

Then, the general solution is:

$$\boxed{y(x) = \left(C_1 + \frac{x}{5}\right)\cos 5x + \left(C_2 - \frac{1}{25}\operatorname{Log}|\cos 5x|\right)\sin(5x)}$$

(9) $mg = 8\ lb. \Rightarrow \quad m = \frac{8\ lb}{g} = \frac{8}{32}\ \frac{lb}{ft/sec^2} = \frac{1}{4}$ slug.

$F = k\Delta\ell.$

$F = k\Delta\ell$ and also $F = mg \Rightarrow k\Delta\ell = mg$

$$\Rightarrow k = \frac{mg}{\Delta\ell} = \frac{8\ lb}{8\ ft} = \underbrace{1\ lb/ft}$$

There is no friction neither external forces.

Newton's law says: $\quad m\ddot{y} + ky = 0 \qquad y(0) = -\frac{1}{2}$ ft

$\frac{1}{4}\ddot{y} + y = 0 \qquad \dot{y}(0) = -\frac{3}{2}$ ft/sec.

$\Rightarrow y(t) = C_1\cos 2t + C_2\sin(2t)$

$\boxed{y(t) = -\frac{1}{2}\cos 2t - \frac{3}{4}\sin 2t}$ feet

(b) $R = \sqrt{C_1^2 + C_2^2} = \sqrt{\frac{1}{4} + \frac{9}{16}} = \frac{\sqrt{13}}{4}$ feet.

$\tan\varphi = \frac{C_2}{C_1} = \frac{-3/4}{-1/2} = \frac{3}{2}$

Since $C_1 < 0$: $\varphi = \arctan\left(\frac{3}{2}\right) + \pi \approx 4.1243 \approx 1.3128\pi$

Then: $\boxed{y(t) = \frac{\sqrt{13}}{4}\cos(2t - 1.3128\pi)} = \frac{\sqrt{13}}{4}\sin(2t + 1.187\pi)$ ~~(Go to page (4))~~.

(c) To find zeros:

Let $\theta = 2t - 1.31\pi = 2(t - 0.65\pi)$. Then, $\cos\theta = 0$

implies: $\theta = -\frac{3\pi}{2} \Rightarrow 2(t - 0.65\pi) = -\frac{3\pi}{2} \dots \Rightarrow t = -\frac{\pi}{10} < 0$

$\theta = -\frac{\pi}{2} \Rightarrow 2(t - 0.65\pi) = -\frac{\pi}{2} \dots \Rightarrow t = \frac{4}{10}\pi > 0$

$\theta = \pi/2 \Rightarrow 2(t - 0.65\pi) = \pi/2 \dots \Rightarrow t = \frac{9}{10}\pi > 0 \leftarrow t.$

$\theta = 3\pi/2 \Rightarrow 2(t - 0.65\pi) = 3\pi/2 \dots \Rightarrow t = \frac{14}{10}\pi > 0$

$= 1\,t =$

$\Rightarrow \quad \theta = \frac{\pi}{2} \quad \Rightarrow \boxed{t_0 = 2.8275 \text{ sec}} = \frac{9}{10}\pi \text{ sec.}$

$$\dot{y}(t) = -\frac{2}{4}\sqrt{13}\sin(2t - 1.31\pi)$$

$$\dot{y}(t_0) = -\frac{\sqrt{13}}{2}\sin\left(\frac{\pi}{2}\right) = -\frac{\sqrt{13}}{2}\text{ feet/sec} < 0$$

The body goes downwards.

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(10) $m = 20gr$. $\Delta L = 5cm$. Use. $g \approx 10 m/sec^2 = 1000 \frac{cm}{sec^2}$

Use the cgs-system. Everything is consistent.

	MKS	cgs
S:	1 second	1 second ← S
M:	1 m	= 100 cm. ← c
K:	1 kg	= 1000 gr ← g
	1 Newton	= 10^5 dynes. $\left(dy = gr \cdot \frac{cm}{sec^2}\right)$.

1 Joule = 10^7 ergs. $\left(erg = gr \cdot \frac{cm^2}{sec^2}\right)$.

$mg = k\Delta \ell \Rightarrow$ Hooke's constant:

$$k = \frac{mg}{\Delta L} = \frac{(20gr)(1000cm/sec^2)}{5cm}$$

$$\Rightarrow \boxed{k = 4000 \text{ gr/sec}^2 = 4000 \text{ dy/cm}}$$

Newton's law: $20\ddot{y} + 400\dot{y} + 400y = 0$

$b = 400 \frac{dy\ sec}{cm}$

$\Rightarrow \ddot{y} + 20\dot{y} + 200y = 0 \quad \begin{cases} y(0) = -2\ cm \\ \dot{y}(0) = 0\ cm/seg. \end{cases}$

("y se suelto").

("... y se suelto" means $\dot{y}(0) = 0$).

Then, $y(t) = e^{rt}$, $r = $ const. $\Rightarrow r^2 + 20r + 200 = 0$.

$$r = \frac{-20 \pm \sqrt{(20)^2 - 4(200)}}{2} \Rightarrow r = -10 \pm 10i$$

$$\Rightarrow \boxed{y(t) = \left(C_1 \cos(10t) + C_2 \sin(10t)\right) e^{-10t}}$$

$$\dot{y}(t) = \left(-10C_1 \sin(10t) + 10C_2 \cos(10t)\right) e^{-10t} + \left(C_1 \cos 10t + C_2 \sin(10t)\right)(-10)e^{-10t}.$$

$y(0) = C_1 = -2\ cm$

$\dot{y}(0) = 10C_2 - 10C_2 = 0 \quad \Rightarrow \boxed{C_1 = C_2 = -2\,cm.}$

Then $\boxed{y(t) = -2\left(\cos 10t + \sin(10t)\right) e^{-10t}\ cm}$

is the solution to the initial value problem.

Now: $A = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ cm. is the amplitude.

$\tan\varphi = \frac{C_2}{C_1} = \frac{-2}{-2} = 1$ Since $C_2 < 0 \Rightarrow \varphi = \text{Arctan}\,1 + \pi$.

$= \frac{\pi}{4} + \pi = \frac{5\pi}{4}$.

$$\Rightarrow \boxed{y(t) = 2\sqrt{2} \cos\left(10t - \frac{5}{4}\pi\right)}$$

Remember:
$$\cos\alpha = \sin\left(\alpha + \frac{\pi}{2}\right)$$

$$y(t) = 2\sqrt{2}\,\sin\left(\left(10t - \frac{5\pi}{4}\right) + \frac{\pi}{2}\right).$$

$$\boxed{y(t) = 2\sqrt{2}\,\sin\left(10t - \frac{\pi}{4}\right)}$$

Note on problem 9, item (b)

We found:
$$y(t) = \frac{\sqrt{13}}{4}\cos(2t - 1.3128\pi$$
$$= \frac{\sqrt{13}}{4}\sin\left(2t - 1.3128\pi + \frac{\pi}{2}\right)$$
$$= \frac{\sqrt{13}}{4}\left(\sin\left(2t - 0.8128\pi\right)\right.$$
$$= \frac{\sqrt{13}}{4}\sin\left(2t - 0.8128\pi + 2\pi\right)$$
$$\boxed{y(t) = \frac{\sqrt{13}}{4}\sin\left(2t + 1.187\pi\right)}$$
