UNIVERSIDAD AUTÓNOMA METROPOLITANA - AZCAPOTZALCO
TRIMESTRE: OTOÑO DE 2020.
CÁLCULO INTEGRAL
EXAMEN GLOBAL (FORMA REMOTA).
FECHA: LUNES 8 DE MARZO DE 2021.
HORA 16:00. HORA DE ENTREGA: 19:00 A 19:30

Nombre: ANSWER KEY

- Si va a presentar algún examen **PARCIAL**, resuelva la parte correspondiente. Cada problema tiene idéntico peso a los demás. Tiene una hora y media (90 minutos) para resolverlo y 30 minutos adicionales para escanearlo y subirlo al *Google Classroom.*
- Si va a presentar el examen **global** resuelva los problemas marcados con una **(G)**. Cada problema indica el puntaje correspondiente. Dispone de 3 horas para resolverlo y 30 minutos adicionales para escanearlo y subirlo al *Google Classroom.*
- El examen es **INDIVIDUAL** y se resuelve de manera **INDIVIDUAL**. Está prohibido recibir ayuda de terceras personas o usar recursos no especificados.
- Puede usar sus libros, apuntes y una calculadora sencilla o graficador sencillo. Cite cuando use libro, apuntes o su calculadora. Si salen fracciones o raíces, NO las convierta a decimales con su calculadora. Déjelas indicadas (a menos que vaya a estimar cantidades físicas).
- **Para recibir puntaje**: Conteste correctamente. Escriba de forma clara y concisa. Entregue su trabajo limpio y con sus ideas en orden. SIMPLIFIQUE y muestre todas sus cuentas. **EXPLIQUE, ARGUMENTE y JUSTIFIQUE** sus respuestas.
- Problema **SIN explicación, desarrollo, justificación o argumento** vale **CERO** puntos.

No olvide elaborar la carátula del examen y anexarla con su examen escaneado.

PROBLEMAS EXAMEN PARCIAL 1:

(1) (a) **(G: 5 puntos)** Usando sumas de Riemann, estime el valor de la siguiente integral, escogiendo los extremos izquierdos de los subintervalos para la evaluación de la función, y dividiendo en 5 subintervalos.

$$\int_1^6 \frac{1}{y^2}\,dy.$$

(b) **(G: 3 puntos)** Sin calcular la integral, diga si la estimación es una sobre-estimación (es mayor) o una sub-estimación (es menor) del valor de la integral. Explique.

(2) **(G: 5 puntos)** Encuentre la derivada de la siguiente función.

$$G(x) = \int_{\sin(\sqrt[3]{x})}^{1+x^2} e^{-y^2}\,dy.$$

(3) Calcule las siguientes integrales.

(a) **(G: 10 puntos)**

$$\int_4^{25} \frac{1}{1+2\sqrt{w}+w}\,dw.$$

(b) **(G: 12 puntos)**

$$\int \left(\operatorname{Log} v - \frac{1}{v}\right)^2\,dv.$$

(c)

$$\int_2^3 \left(\frac{\operatorname{Log} u}{u}\right)^3\,du.$$

PROBLEMAS EXAMEN PARCIAL 2:

Calcule las siguientes integrales:

(1)
$$\int \tan^3(4x)\sec^3(4x)\,dx.$$

(2) De la forma de la descomposición en fracciones parciales de la siguiente función racional. ¿Es necesario efectuar una división polinomial previamente?
$$\frac{x^{10}+x^5+1}{(x^2+4x+4)^3(x^4+16)^4}$$

(3) **(G: 15 puntos)**
$$\int \frac{x^3}{\sqrt{2-4x^2}}\,dx.$$

(4) **(G: 15 puntos)**
$$\int \frac{-2x^2+x+2}{x^3+x}\,dv.$$

(5) **(G: 10 puntos)**
$$\int_0^\infty 9xe^{-x/9}\,dx.$$

PROBLEMAS EXAMEN PARCIAL 3:

(1) **(G: 10 puntos)** Calcule el área de la región limitada por las curvas $y=3^{1-x}$ y $y=3x-1$ para x entre 0 y 1.

(2) **(G: 15 puntos)** Calcule el volumen del sólido de revolución obtenido al rotar, alrededor del eje Y, la región acotada por las curvas $y=\sin x$, $y=\cos x$ y $x=0$.

(3) Calcule, analíticamente (usando integrales), la longitud de la gráfica de la función $f(x)=\sqrt{9-x^2}$ para x entre -3 y 3. Posteriormente, compare con sus conocimientos de geometría plana.

UAM - AZCAPOTZALCO | CÁLCULO INTEGRAL | Lunes, 8 de marzo de 2021.

EXAMEN GLOBAL ANSWER KEY ~~EXAMEN PARCIAL #1~~

(1) We know that.

(a) $\displaystyle\int_1^6 \frac{1}{y^2}\,dy = \lim_{N\to\infty} \sum_{i=1}^{N} \frac{1}{x_i^{*2}} \Delta x_i$,

where: $\{x_i\}_{i=0}^{N}$ is a partition of $[1,6]$, and $x_i^* \in [x_{i-1}, x_i]$

and $\Delta x_i \longrightarrow 0$ as $N \longrightarrow \infty$

For Δx fixed, $\Delta x = \frac{b-a}{N} = \frac{6-1}{N} = \frac{5}{N}$.

and $x_i^* \in [x_{i-1}, x_i]$, $x_i = x_0 + i\Delta x$, $x_0 = 1$ and $x_N = 6$.

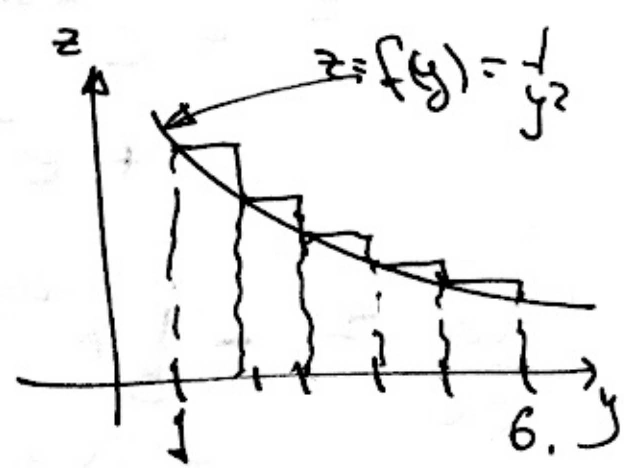
If $N = 5$; $\Delta x = 1$, and $x_i^* = x_{i-1}$, $i = 1,2,3,4,5$.

ie $x_i^* = 1, 2, 3, 4, 5$; $(x_i^* = x_{i-1} = x_0 + (i-1)\Delta x,\ i = 1,2,3,4,5)$

$$\int_1^6 \frac{1}{y^2}\,dy \approx \sum_{i=1}^{N} \frac{1}{(x_{i-1})^2}\Delta x = \frac{1}{1} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2}$$

$$= 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} = 1.4636111\ldots$$

(b) Since we are taking left-side sums, we are over the graph of $f(y) = \frac{1}{y^2}$, so that this integral is an overestimate



$= ($

② We use the Fundamental Theorem of Calculus and the Chain Rule (and some other properties of the Integral:

$$\frac{dG}{dx} = \frac{d}{dx}\int_{\sin\sqrt[3]{x}}^{1+x^2} e^{-y^2}\,dy =$$

$$= \frac{d}{dx}\left(\int_{\sin\sqrt[3]{x}}^{0} e^{-y^2}\,dy + \int_{0}^{1+x^2} e^{-y^2}\,dy\right)$$

$$= \frac{d}{dx}\left(-\int_{0}^{u(x)} e^{-y^2}\,dy + \int_{0}^{v(x)} e^{-y^2}\,dy\right)$$

where $u(x) = \sin\sqrt[3]{x}$, $v(x) = 1+x^2$

By the F.T.C. and the Chain rule:

$$= -e^{-u^2(x)}\frac{du}{dx} + e^{-v^2(x)}\frac{dv}{dx}$$

$$= -e^{-\sin^2(x^{1/3})} \cdot \cos(x^{1/3})\left(\frac{1}{3}\right)x^{-2/3}$$

$$+ e^{-(1+x^2)}\,2x$$

i.e

$$\boxed{\frac{dG}{dx} = -\frac{1}{3}x^{-2/3}e^{-\sin^2(x^{1/3})}\cos(x^{1/3}) + 2x e^{-(1+x^2)}.}$$

3(a)

(a) $\int_4^{25} \frac{1}{1+2\sqrt{w}+w}\,dw = \int_4^{25} \frac{1}{(1+\sqrt{w})^2}\,dw$

Let $y=\sqrt{w} \Rightarrow w = y^2 \Rightarrow \frac{dw}{dy} = 2y.$

If $w=4 \Rightarrow y=2$
$w=25 \quad y=5.$

and so:

$$= \int_2^5 \frac{1}{(1+y)^2}\,2y\,dy = 2\int_2^5 \frac{1+y-1}{(1+y)^2}\,dy$$

$$= 2\int_2^5 \left(\frac{1}{(1+y)} - \frac{1}{(1+y)^2}\right) dy = 2\left(\log(1+y) + \frac{1}{(1+y)}\right)\Bigg|_2^5$$

$$= 2\left[\log\left(\frac{6}{3}\right) + \frac{1}{6} - \frac{1}{3}\right].$$

Then. $\boxed{\int_4^{25} \frac{1}{1+2\sqrt{w}+w}\,dw = 2\left(-\frac{1}{6}+\log 2\right)}$

③(b) $\int \left(\text{Log}\, v - \frac{1}{v}\right)^2 dv =$

$= \int \text{Log}^2 v - 2\,\frac{\text{Log}\, v}{v} + \frac{1}{v^2}\, dv$

$= \int \text{Log}^2 v\, dv - 2\int \frac{d}{dv}\left(\text{Log}^2 v\right) dv + \int \frac{1}{v^2}\, dv$

$= \int \text{Log}^2 v\, dv - \text{Log}^2 v - \frac{1}{v}$

It remains to compute, by $y = \text{Log}\, v \Rightarrow v = e^y$, $\frac{dv}{dy} = e^y$

$\int (\text{Log}^2 v)\, dv = \int y^2 e^y\, dy$

$= y^2 e^y - \int 2y e^y\, dy = y^2 e^y - 2y e^y + \int 2e^y\, dy$

$= y^2 e^y - 2y e^y + 2e^y = (y^2 - 2y + 2)\, e^y$

$= (\text{Log}^2 v - 2\,\text{Log}\, v + 2)\, v$

Hence

$$\boxed{\int \left(\text{Log}\, v - \frac{1}{v}\right)^2 dv = v\left(\text{Log}^2 v - 2\,\text{Log}\, v + 2\right) - \text{Log}^2 v - \frac{1}{v} + C.}$$

③ (c) $\int_2^3 \frac{\log^3 u}{u^3}\,du = \int_2^3 \frac{\mathrm{Log}^3 u}{u^2}\,\frac{1}{u}\,du.$

$y = \log u \qquad \frac{dy}{du} = \frac{1}{u}\,, \quad u = e^y$

$= \int \frac{y^3}{e^{2y}}\,dy = \int y^3 e^{-2y}\,dy$ Integrating by parts:

$= y^3 \frac{e^{-2y}}{-2} - \int \frac{3y^2}{-2}\frac{e^{-2y}}{(-2)}\,dy$

$= y^3 \frac{e^{-2y}}{-2} - \frac{3y^2}{}\frac{e^{-2y}}{(-2)^2} + \int 6y\,\frac{e^{-2y}}{(-2)^2}\,dy$

$= y^3 \frac{e^{-2y}}{-2} - 3y^2\frac{e^{-2y}}{(-2)^2} + 6y\frac{e^{-2y}}{(-2)^3} - \int 6\frac{e^{-2y}}{(-2)}\,dy$

$= y^3 \frac{e^{-2y}}{-2} - 3y^2\frac{e^{-2y}}{(-2)^2} + 6y\frac{e^{-2y}}{(-2)^3} - 6\frac{e^{-2y}}{(-2)^4} + C$

$= \left[\frac{y^3}{(-2)} - \frac{3y^2}{(-2)^2} + \frac{6y}{(-2)^3} - \frac{6}{(-2)}\right] e^{-2y} + C$

$$\boxed{= \left(-\frac{1}{2}\right)\left[\log^3 u + \frac{3}{2}\log^2 u + \frac{3}{2}\log u - 6\right]\frac{1}{u^2}\Bigg|_2^3 + C}$$

EXAMEN PARCIAL #2

1. $$\int \tan^3(4x)\sec^3(4x)\,dx = \frac{1}{4}\int \tan^3 y \sec^3 y\, dy$$

$y = 4x.$

$$= \frac{1}{4}\int \tan y \sec^2 y(\tan y \sec y)\,dy = \frac{1}{4}\int \tan^2 y \sec^2 y \frac{d}{dy}(\sec y)\,dy$$

$$= \frac{1}{4}\int (\sec^2 y - 1)(\sec^2 y)\frac{d}{dy}(\sec y)\,dy$$

$u = \sec y$

$$= \frac{1}{4}\int (u^2-1)\,u^2\,du = \frac{1}{4}\int u^4 - u^2\,du$$

$$= \frac{1}{4}\left(\frac{u^5}{5} - \frac{u^3}{3}\right) + C = \frac{1}{4}\left(\frac{\sec^5 y}{5} - \frac{\sec^3 y}{3}\right) + C$$

thus:

$$\boxed{\int \tan^3(4x)\sec^3(4x)\,dx = \frac{1}{4}\left(\frac{\sec^5(4x)}{5} - \frac{\sec^3(4x)}{3}\right) + C}$$

(2) We have the rational function.

$$\frac{x^{10}+x^5+1}{(x^2+4x+4)^3(x^4+16)^4}. \qquad \begin{aligned} Deg(\text{Numerator}) &= 10 \\ Deg(\text{denominator}) &= 2\cdot 3+4\cdot 4 \\ &= 22. \end{aligned}$$

Then, it is <u>not</u> required polynomial division.

Now: let us factor the denominator.

$$(x^2+4x+4)^3(x^4+16)^4 = ((x+2)^2)^3(x^4+8x^2+16-8x^2)^4$$

$$= (x+2)^6((x^2+4)^2-8x^2)^4 = (x+2)^6\left((x^2+4-\sqrt{8}x)(x^2+4+\sqrt{8}x)\right)^4$$

$$=(x+2)^6(x^2-\sqrt{8}x+4)^4(x^2+\sqrt{8}x+4)^4.$$

Then:

$$\frac{x^{10}+x^5+1}{(x+2)^6(x^2-\sqrt{8}x+4)^4(x^2+\sqrt{8}x+4)^4} =$$

$$= \frac{A_1}{x+2}+\frac{A_2}{(x+2)^2}+\frac{A_3}{(x+2)^3}+\frac{A_4}{(x+2)^4}+\frac{A_5}{(x+2)^5}+\frac{A_6}{(x+2)^6}+$$

$$\frac{B_1x+C_1}{(x^2-\sqrt{8}x+4)}+\frac{B_2x+C_2}{(x^2-\sqrt{8}x+4)^2}+\frac{B_3x+C_3}{(x^2-\sqrt{8}x+4)^3}+\frac{B_4x+C_4}{(x^2-\sqrt{8}x+4)^4}+$$

$$+\frac{D_1x+E_1}{(x^2+\sqrt{8}x+4)}+\frac{D_2x+E_2}{(x^2+\sqrt{8}x+4)^2}+\frac{D_3x+E_3}{(x^2+\sqrt{8}x+4)^3}+\frac{D_4x+E_4}{(x^2+\sqrt{8}x+4)^4}$$

③ $\int \frac{x^3}{\sqrt{2-4x^2}}\,dx$. Trigonometric substitution:

$x = \frac{1}{\sqrt2}\sin\theta$, $\frac{dx}{d\theta} = \frac{\cos\theta}{\sqrt2}$

$$= \frac{1}{2}\int \frac{x^3}{\sqrt{\frac12 - x^2}}\,dx = \frac{\sqrt{1-\sin^2\theta}}{\sqrt2} = \sqrt{\frac12 - x^2}$$

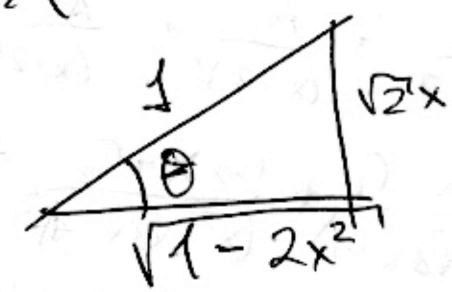
$$= \frac12 \cdot \frac{1}{2^{3/2}}\int \sin^3\theta \, d\theta \qquad \frac{dx}{\sqrt{\frac12 - x^2}} = d\theta.$$

$$= \frac{1}{2\cdot 2^{2/3}}\int \sin^2\theta \sin\theta \, d\theta = \frac{1}{4\sqrt2}\int \sin^2\theta \frac{d}{d\theta}(-\cos\theta)\, d\theta$$

$$= \frac{-1}{4\sqrt2}\int (1-\cos^2\theta)\frac{d}{d\theta}(\cos\theta)\, d\theta = \frac{-1}{4\sqrt2}\int (1-y^2)\, dy$$

$y = \cos\theta$

$$= \frac{-1}{4\sqrt2}\left(y - \frac13 y^3\right) = \frac{-1}{4\sqrt2}\left(\cos\theta - \frac13\cos^3\theta\right) + C$$



Now: $\sin\theta = \sqrt2 x$

Then $\cos\theta = \sqrt{1-2x^2}$

$$\cos^2\theta = 1 - \sin^2\theta = 1 - 2x^2$$

Then:

$$\boxed{\int \frac{x^3}{\sqrt{2-4x^2}}\,dx = \frac{-1}{4\sqrt2}\sqrt{1-2x^2}\left(\frac23 - \frac23 x^2\right) + C}$$

④ $\displaystyle\int \frac{-2x^2+x+2}{x^3+x}\,dx$. Since $\deg(\text{Numerator}) = 2$, $\deg(\text{Denominator}) = 3$

No polynomial division. Then, partial fractions.

$$\frac{-2x^2+x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)x}{x(x^2+1)}.$$

Then $-2x^2+x+2 = A(x^2+1) + (Bx+C)x$

Take $x=0$| $\quad 2 = A + 0 \Rightarrow A = 2.$

$x=1$| $\quad -2+1+2 = 2A + (B+C)$

$x=-1$| $\quad -2-1+2 = 2A + (B-C)$

Then, previous two eq's:

$$1 = 4 + (B+C) \Rightarrow B+C = -3$$
$$-1 = 4 + (B-C) \Rightarrow +B-C = -5:$$

Then $2B = -8 \Rightarrow \underline{B=-4}$| We therefore can conclude:

$2C = 2, \Rightarrow \underline{C = 1}$|

$$\int \frac{-2x^2+x+2}{x^3+x}\,dx = \int \frac{2}{x} + \frac{(-4x)+1}{x^2+1}\,dx.$$

$$= \int \frac{2}{x} + \frac{(-4)x}{x^2+1} + \frac{1}{x^2+1}\,dx$$

$$= \boxed{2\log|x| - 2\log(x^2+1) + \arctan x + C}$$

⑤ $\int_0^{\infty} 9xe^{-x/9}\,dx = \lim_{M\to\infty} \int_0^{M} 9x e^{-x/9}\,dx.$

Solve first the definite integral:

$$\int_0^M 9x\, e^{-x/9}\,dx = 9^3 \int_0^{M/9} y e^{-y}\,dy =$$

$$y = \frac{x}{9}.$$

$$= 9^3\left[y\frac{e^{-y}}{-1}\Big|_0^{M/9} - \int_0^{M/9} \frac{e^{-y}}{-1}\,dy\right] = 9^3\left[\frac{ye^{-y}}{-1} - e^{-y}\right]\Big|_0^{M/9}$$

$$= -9^3\left[\left(\frac{M}{9}e^{-M/9} + e^{-M/9}\right) - (0+1)\right]$$

L'Hôpital

Now $\lim_{M\to\infty} Me^{-M/9} = \lim_{M\to\infty} \frac{M}{e^{M/9}} = \lim_{M\to\infty} \frac{1}{\frac{1}{9}e^{M/9}} = 0$

thus:

$$\int_0^{\infty} 9xe^{-x/9}\,dx = \lim_{M\to\infty} (-9^3)\left[\left(\frac{M}{9}e^{-M/9} + e^{-M/9}\right) - 1\right].$$

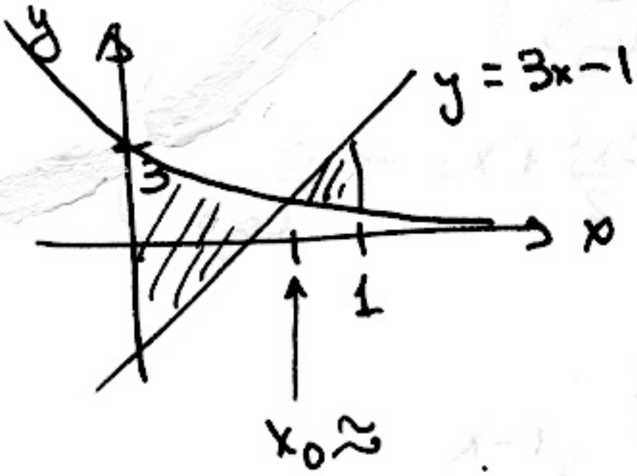
$$= 0 + 0 + 9^3.$$

i.e. $\boxed{\int_0^{\infty} 9xe^{-x/9}\,dx = 9^3}$

EXAMEN PARCIAL 3

① Ploting the curves $y = 3^{1-x} = \frac{3}{3^x} = f(x)$

and $y = 3x - 1. = g(x)$



$f(1) = 1$, $g(1) = 2$

The intersection point is approximately: $x_0 \approx 0.765$ (Geogebra)

Then

$$\text{Area} = \int_0^1 |f(x) - g(x)| dx = \int_0^{x_0} (f(x) - g(x)) dx + \int_{x_0}^1 (g(x) - f(x)) dx$$

So, we need:

$$\int f(x) - g(x) dx = \int 3 \cdot 3^{-x} - 3x + 1 dx$$

$$= 3 \int 3^{-x} dx - \frac{3x^2}{2} + x. =$$

$\lfloor 3^{-x} = e^{-y}$

$-x \log 3 = -y$

$y = (\log 3) x.$

$\frac{dy}{dx} = \log 3$

$$= \frac{3}{\log 3} \int e^{-y} dy - \frac{3x^2}{2} + x.$$

$$= \frac{-3}{\log 3} e^{-y} - \frac{3x^2}{2} + x$$

$$= \frac{-3}{\log 3} 3^{-x} - \frac{3}{2} x^2 + x.$$

=//=

$$\text{Area} = \left(\frac{-3^{1-x}}{\log 3} - \frac{3}{2}x^2 + x\right)\Bigg|_0^{x_0} + \left(\frac{3x^2}{2} - x + \frac{3^{1-x}}{\log 3}\right)\Bigg|_{x_0}^{1}$$

$$= \left(\frac{-3^{1-x_0}}{\log 3} - \frac{3}{2}x_0^2 + x_0 + \frac{3}{\log 3}\right)$$

$$+ \left(\frac{1}{2} + \frac{1}{\log 3} - \frac{3x_0^2}{2} + x_0 - \frac{3^{1-x_0}}{\log 3}\right)$$

Since: $3^{1-x_0} = 3x_0 - 1$;

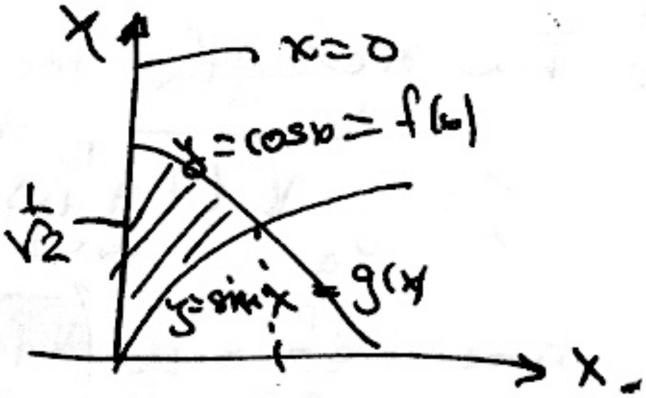
$$= \frac{1}{2} + \frac{4}{\log 3} - 3x_0^2 + 2x_0 - 2\frac{^{1-x_0}}{\log 3}$$

$$= \frac{1}{2} + \frac{4}{\log 3} - 3x_0^2 + 2x_0 - 2\frac{(3x_0 - 1)}{\log 3}$$

$$\approx \frac{1}{2} + \frac{4}{\log 3} - 3x_0^2 + 2x_0 - \frac{2(1.25)}{\log 3}$$

$$\approx \frac{1}{2} + \frac{1.5}{\log 3} - 3x_0^2 + 2x_0 \approx 2.47$$

(2) Thee bounded regime is:



Then, we can use the shells method.

We first compute the volume by rotating $y = f(x) = \cos x$

$$V_1 = 2\pi \int_0^{\pi/4} x\, f(x)\, dx = 2\pi \int_0^{\pi/4} x \cos x\, dx$$

$$= 2\pi \left(x \sin x \Big|_0^{\pi/4} - \int_0^{\pi/4} \sin x\, dx \right)$$

$$= 2\pi \left(x \sin x + \cos x \right) \Big|_0^{\pi/4} = 2\pi \left[\left(\frac{\pi}{4}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) - (1) \right]$$

$$= 2\pi \left[\frac{\pi}{4}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 \right].$$

The second volume is generated by $y = \sin x$.

$$V_2 = 2\pi \int_0^{\pi/4} x \sin x\, dx = 2\pi \left[-x \cos x \Big|_0^{\pi/4} + \int_0^{\pi/4} \cos x\, dx \right]$$

$$= 2\pi \left[-x \cos x + \sin x \right] \Big|_0^{\pi/4} = 2\pi \left[-\frac{\pi}{4}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right]$$

Then, the volume is

$$\boxed{V = V_1 - V_2 = 2\pi \left[\frac{\pi \sqrt{2}}{4} - 1 \right].}$$

③ The arclength for the function $y = f(x)$ is.

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx.$$

Here $\frac{df}{dx} = \frac{d}{dx}\left(\sqrt{9-x^2}\right) = \frac{-x}{\sqrt{9-x^2}}$

Then

$$L = \int_{-3}^{3} \sqrt{1+\frac{x^2}{9-x^2}}\,dx = \int_{-3}^{3} \frac{\sqrt{9}}{\sqrt{9-x^2}}\,dx.$$

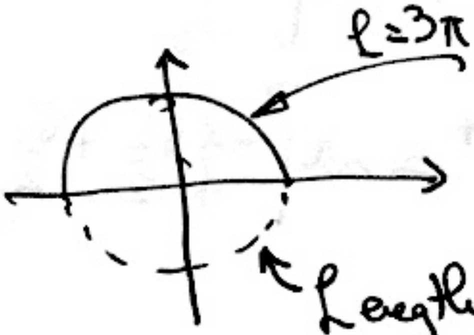
$x = 3\sin\theta$

$$= 3\int_{-3}^{3} \frac{1}{\sqrt{9-x^2}}\,dx = 3\int_{-\pi/2}^{\pi/2} \frac{1}{3\cos\theta}\,3\cos\theta\,d\theta$$

$$= 3\int_{-\pi/2}^{\pi/2} d\theta = 3\,\theta\Big|_{-\pi/2}^{+\pi/2} = 3\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)$$

$$= 3\pi$$

Now, $f(x) = \sqrt{9-x^2}$ is a semi-circle of radius $r = 3$. The length of the full circle is



Length of circle $= 2\pi R$
$= 2\pi \cdot 3 = 6\pi$

Length circle $= 2\pi R$
$= 2\pi 3 = 6\pi$

$\boxed{\text{Half-circle} = 3\pi}$

Both coincide.