UAM-AZCAPOTZALCO. CÁLCULO INTEGRAL II Friday, December 9, 2022

Examen #2 SOLUTION KEY

① Here, we make the substitution. $y = 3x + x^3$

Then $\frac{dy}{dx} = 3 + 3x^2 = 3(1+x^2)$.

Hence, the substitution rules says:

$$\int \frac{1+x^2}{\sqrt{3x+x^3}}\,dx = \int \frac{1}{\sqrt{y}}\frac{dy}{3} = \frac{2}{3}\sqrt{y} + C$$

$$= \boxed{\frac{2}{3}\sqrt{3x+x^3} + C}$$

② $\int \ln s\, s^5\, ds = \int \ln s \left(\frac{s^6}{6}\right)' ds$, and integrating by parts

$$= (\ln s)\frac{s^6}{6} - \int (\ln s)' \frac{s^6}{6}\,ds = \frac{s^6}{6}\ln s - \int \frac{1}{s}\frac{s^6}{6}\,ds$$

$$= \frac{s^6}{6}\ln s - \int \frac{s^5}{6}\,ds = \frac{s^6}{6}\ln s - \frac{s^6}{6\cdot 6} + C$$

$$\boxed{\int (\ln s)\, s^5\, ds = \frac{s^6}{36}(6\ln s - 1) + C}$$

③ $\int \cos^2 x \sin^5 x\, dx = \int \cos^2 x \sin^4 x \sin x\, dx =$

$u = \cos x$

$$= \int \cos^2 x\,(1-\cos^2 x)^2 (-\cos x)'\,dx = -\int u^2(1-u^2)^2\,du$$

$$= -\int u^2 - 2u^4 + u^6\, ds = -\left(\frac{u^3}{3} - \frac{2}{5}u^5 + \frac{u^7}{7}\right) + C = -\left(\frac{\cos^3 x}{3} - \frac{2\cos^5 x}{5} + \frac{\cos^7 x}{7}\right) + C$$

④ $\displaystyle\int \frac{1}{x^2\sqrt{x^2+9}}\,dx \qquad x = 3\tan\theta \qquad \frac{dx}{d\theta} = 3\sec^2\theta$

$$\sqrt{x^2+9} = 3\sqrt{\tan^2\theta+1} = 3|\sec\theta| = 3\sec\theta \qquad \theta\in\left(-\tfrac{\pi}{2},\tfrac{\pi}{2}\right)$$

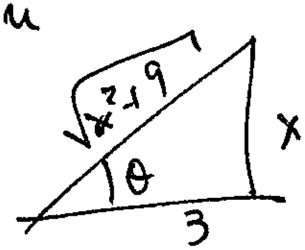
$$= \int \frac{1}{9\tan^2\theta\, 3\sec\theta}\, 3\sec^2\theta\, d\theta = \frac{1}{9}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$= \frac{1}{9}\int \frac{\cos^2\theta}{\sin^2\theta}\,\frac{1}{\cos\theta}\,d\theta = \frac{1}{9}\int \frac{1}{\sin^2\theta}\cos\theta\,d\theta = \frac{1}{9}\int\frac{1}{\sin^2\theta}(\sin\theta)'\,d\theta$$

$u = \sin\theta$

$$= \frac{1}{9}\int \frac{1}{u^2}\,du = -\frac{1}{9u} + C = -\frac{1}{9\sin\theta} + C.$$

$\tan\theta = \frac{x}{3} \implies$



$\implies \sin\theta = \frac{x}{\sqrt{x^2+9}}.$

$$\Rightarrow \boxed{\int \frac{1}{x^2\sqrt{x^2+9}}\,dx = -\frac{1}{9}\,\frac{\sqrt{x^2+9}}{x} + C}$$

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⑤ From $\displaystyle\int_1^7 f(x)\,dx = \int_1^9 f(x)\,dx + \int_9^7 f(x)\,dx = \int_1^9 f(x)\,dx - \int_7^9 f(x)\,dx$

$$= 12 - 12 = 0$$

Then $\displaystyle 2\int_7^9 f(x)\,dx - 3\int_7^9 h(x)\,dx + \int_1^7 f(x)\,dx =$

$$= 2(12) - 3(-4) + 0 = 24 + 12 = 36$$
